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The following research was carried on, at the Massachusetts Institute of Technology, under an appropriation from the American Telegraph and Telephone Co. during the year 1914–1915. The experimental work was carried out at Pierce Hall, Harvard University.

This research constitutes a continuation and extension of that reported to the Academy in September 1912, under the title of "The Impedance of Telephone Receivers, as affected by the Motion of their Diaphragms," [1] by A. E. Kennelly and G. W. Pierce. In that paper of 1912, it was shown that the impedance of a telephone receiver is different, when the diaphragm is free to vibrate, from that which it offers when the diaphragm's motion is damped or prevented. The difference between the "free" impedance as the frequency is varied, and the "damped" impedance, is called the "motional impedance," and measures the velocity of the diaphragm's vibration. When plotted vectorially, this "motional impedance" is found to be a circle passing through the origin of coordinates, and with its diameter depressed through a certain angle. Every telephone receiver and diaphragm possesses its own characteristic motional-impedance circle. The characteristics of this circle, in regard to diameter, depression angle, and distribution of frequency positions, determine certain electrical and mechanical properties of the instrument. Examples of such circle diagrams appear in this paper in Figures 4 and 14.

It was shown in the 1912 paper above referred to, that there are four constants of an ordinary telephone receiver which determine the essentials of its behavior, both electrically and mechanically, throughout the range of ordinary telephonic frequencies (100 to 2500 ∾).

If we consider the impedance of a telephone receiver with the diaphragm prevented from vibrating, and thus incapable of reacting electromagnetically on the coils, when the latter are excited by alternating current, we find that, as might be expected, the impedance of

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[1] See Bibliography, No. 11.

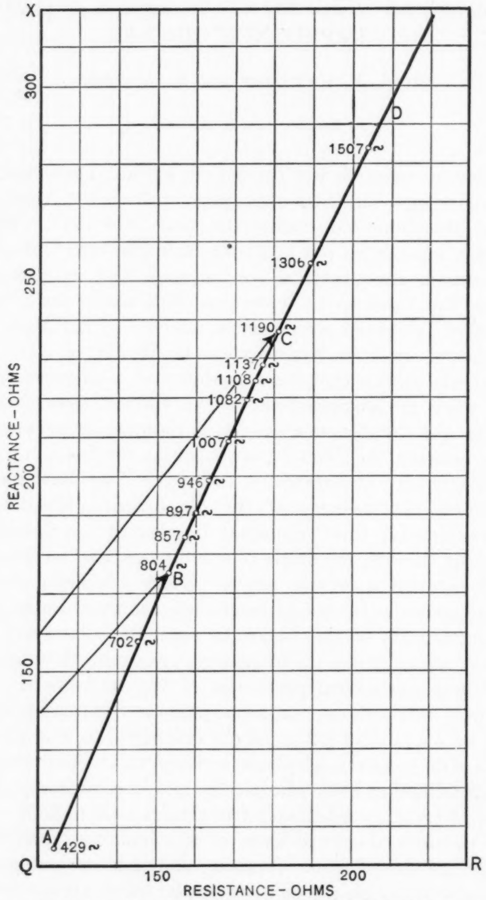

FIG. 1. — LOCUS OF VECTOR DAMPED IMPEDANCE FOR A PARTICULAR RECEIVER.

these coils increases when the frequency is increased, although not in direct proportion. Thus, Figure 1 represents the locus of the impedance of a particular receiver when measured by Rayleigh bridge, with constant sinusoidal alternating-current strength, but with varying frequency. Apparent resistances are measured parallel to the axis QR, and apparent reactances parallel to the axis QX. As the frequency of the exciting alternating current is increased from 400 ∾ to 2500 ∾, the locus of the impedance is the curve A B C D. Thus at 804 ∾, the impedance is found to be OB, and at 1190 ∾, it is increased to OC, where O is the origin (not shown). The increase in apparent resistance is due to increasing power loss at rising frequencies. The increase in reactance $j\,X = j\,L\omega$ ohms, is to be attributed to increase in the angular velocity $\omega = 2\,\pi\,f$ radians per second; where f is the frequency. The apparent inductance L henrys actually diminishes as the frequency rises; so that the increase in the reactance OX is less rapid than the increase in frequency. The impedance measured in this manner is called the "damped impedance."

If now the measurement of the impedance is repeated over the same range of frequency but with the diaphragm released, so as to be able to react electromagnetically on the winding, the locus is found to be a looped curve, such as is represented in Fig. 2; so that one and the same vector impedance OP is found at two distinctly different frequencies. Thus at 702 ∾, the "free impedance" has a magnitude of $153 + j\,161.5$ ohms, and at 1050 ∾, $148 + j\,144$ ohms. This peculiar dual valued behavior of the free impedance is due to the motion of the diaphragm.

If at each of a series of successive rising frequencies, we subtract the vector damped impedance from the vector free impedance, as indicated in Figure 3, it is found that the successive vector differences, due to the motion of the diaphragm, and, therefore, called "motional impedances," lie approximately on a certain circular locus as shown in Figure 4. This circular locus is called the motional-impedance circle for the particular instrument.

The same facts may also be presented in a scalar or non-vector diagram, Figure 5. Here the abscissas represent impressed frequency. The ordinates represent, to the left-hand scale, the magnitude or "vector modulus" of the telephone-receiver impedance. To the right-hand scale, they represent the phase angle or "argument." Heavy curves represent measurements of free impedance, and broken curves corresponding measurements of damped impedance. The measurements made on the particular receiver C, referred to in Figures 1 to 5, are recorded in Table I.

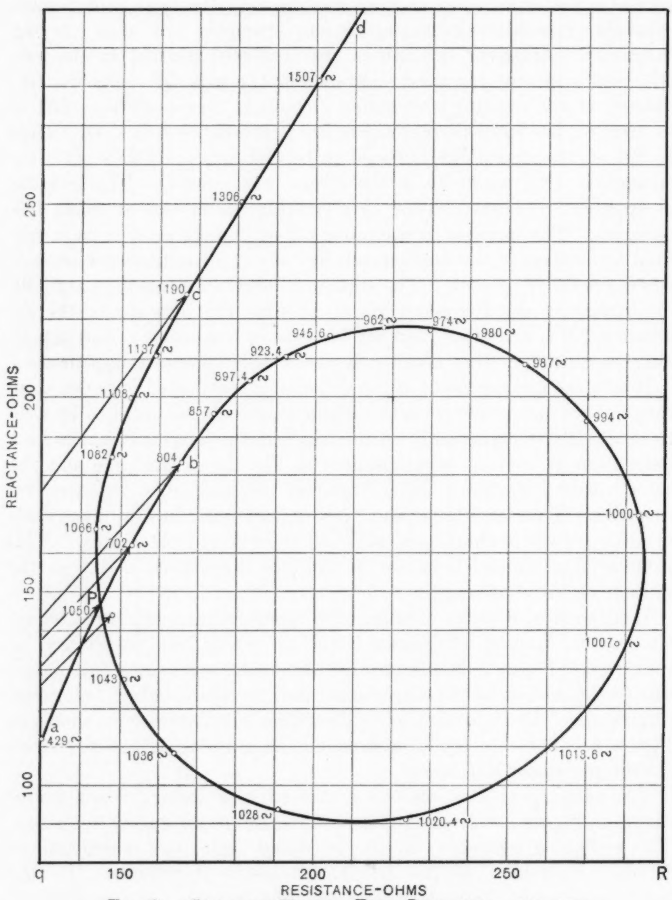

FIG. 2. — LOCUS OF VECTOR FREE IMPEDANCE, FOR THE SAME RECEIVER.

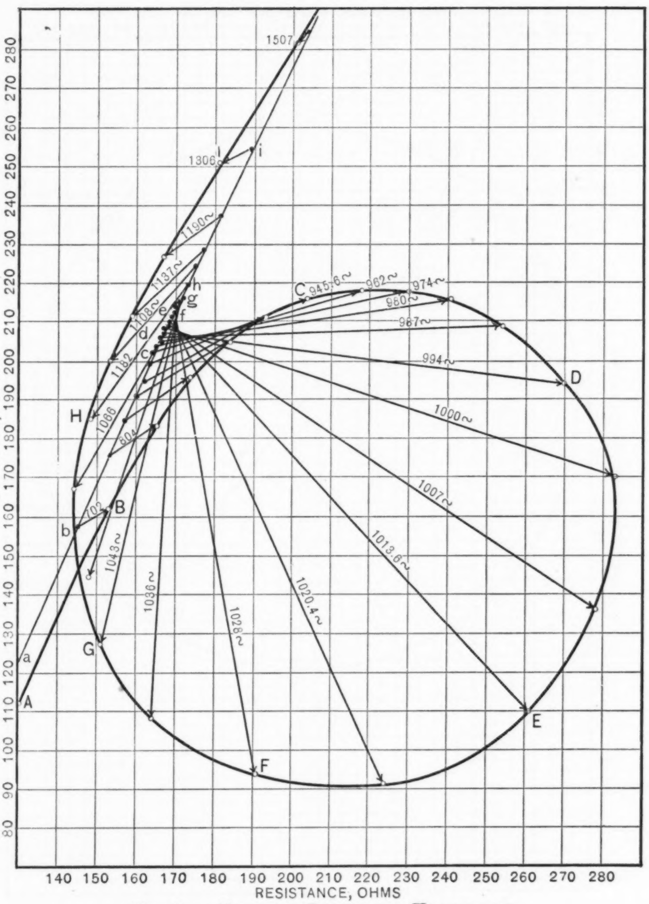

Fig. 3. — Motional Impedance Vectors at Successive Observed Frequencies.

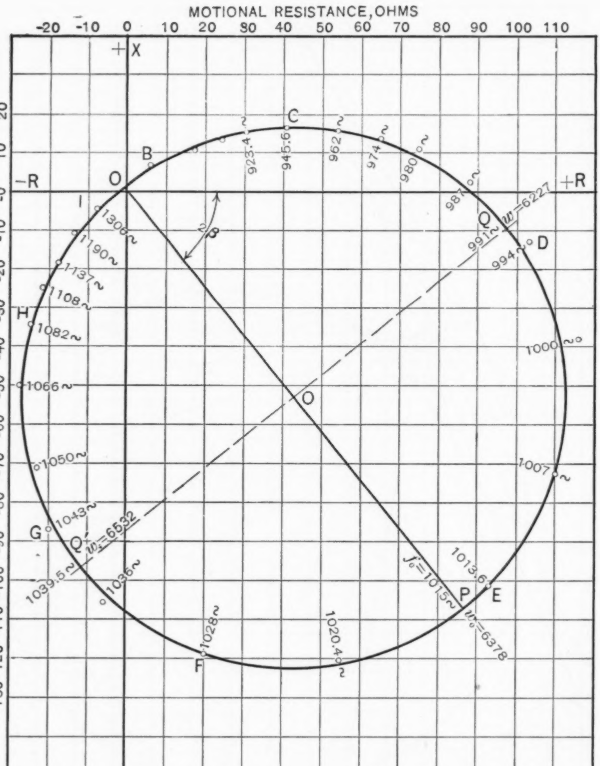

FIG. 4. — MOTIONAL IMPEDANCE CIRCLE OF SAME RECEIVER FROM VECTORS IN FIG. 3.

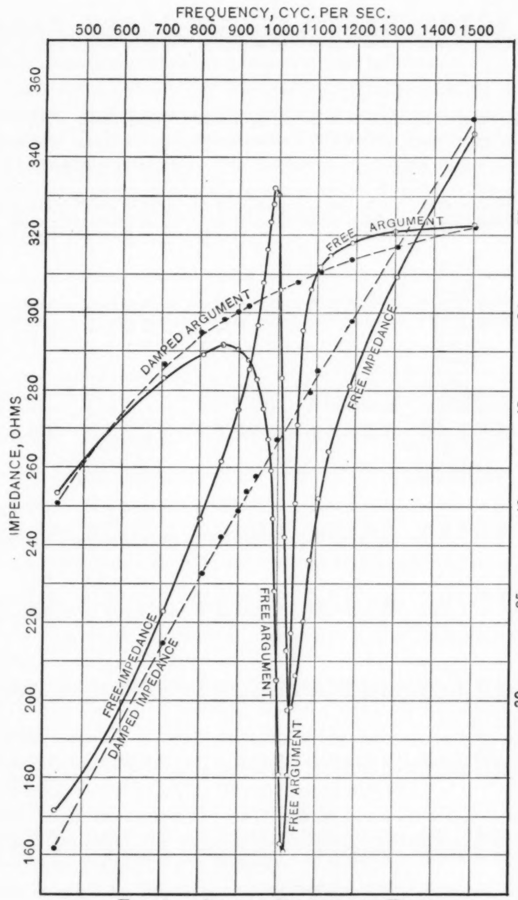

Fig. 5. — Scalar Diagram of Free and Damped Impedance.

TABLE I.

Freq. c. p. s. ∾	Ang. Veloc. ω	R^1 Res. Free ohms	R Res. Damped ohms	R^1–R Motional Res. ohms	L^1 Inductance Free millihenrys	L Inductance Damped millihenrys	L^1–L Motional Inductance millihenrys	X^1 Motional React. ohms $(L^1-L)\omega$	x Max. Cyclic Displacements microns observed	$\dot{x}$ Max. Cyclic Vibration Velocity kines ωx
429	2694	130	124	6	41.5	38.7	2.8	7.5	1.8	0.49
702	4410	153	145	8	36.6	35.7	0.9	4.0	1.6	0.70
804	5052	166	153	13	36.2	34.7	1.5	7.6	1.9	0.96
857	5385	174	157	17	36.3	34.2	2.1	11.3	2.6	1.40
897.4	5640	184	160	24	36.2	33.8	2.4	13.5	2.6	1.47
923.4	5801	193	162	31	36.3	33.6	2.7	15.7	2.85	1.65
945.6	5941	204	163	41	36.3	33.5	2.8	16.6	3.8	2.26
962	6046	218	164	54	36.0	33.4	2.6	15.7	4.56	2.76
974	6118	230	165	65	35.5	33.3	2.2	13.5	4.63	2.83
980	6156	241	166	75	35.0	33.2	1.8	11.1	5.07	3.12
987	6200	254	166	88	33.6	33.2	0.4	2.5	6.02	3.73
994	6245	270	167	103	31.0	33.1	−2.1	−13.1	6.97	4.35
1000	6283	283	167	116	27.0	33.1	−6.1	−38.3	8.23	5.17
1007	6328	278	168	110	21.5	33.0	−11.5	−72.8	9.18	5.81
1013.6	6370	261	168	93	17.2	33.0	−15.8	−101.0	10.1	6.43
1020.4	6412	224	168.5	55	14.2	32.9	−18.7	−119.8	10.45	6.70
1028	6461	191	170	21	14.5	32.8	−18.3	−118.3	9.38	6.06
1036	6508	164	170	−6	16.6	32.8	−16.2	−105.4	8.24	5.36
1043	6552	151	171	−20	19.4	32.7	−13.3	−87.1	7.60	4.98
1050	6597	148	171	−23	21.8	32.6	−10.8	−71.2	6.15	4.06
1066	6698	144	172	−28	25.0	32.5	−7.5	−50.2	4.75	3.18
1082	6798	148	173	−25	27.3	32.4	−5.1	−34.7	3.80	2.58
1108	6966	153	175	−22	28.7	32.2	−3.5	−24.4	2.22	1.55
1137	7145	159	177	−18	29.5	32.0	−2.5	−17.9	0.95	0.68
1190	7478	167	181	−14	30.3	31.7	−1.4	−10.5		
1306	8203	181	189	−8	30.5	31.0	−0.5	−4.1		
1507	9470	201	204	−3	29.7	30.0	−0.3	−2.8		

Mechanics of the Receiver.

The four essential and characteristic constants of any receiver, from the electromechanical point of view, are as follows:

A, the "force factor," or the electromagnetic pull, in dynes on the diaphragm, per absampere [2] of alternating current, subject to the restrictions that A varies somewhat with the frequency and strength of the actuating current. The factor is supposed to be measured at a standard frequency, and with feeble currents (less than 3 milliamperes).

m, the "equivalent mass" of the diaphragm, or the mass, in grams,

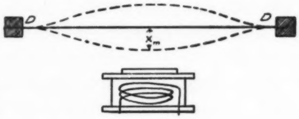

Fig. 6.

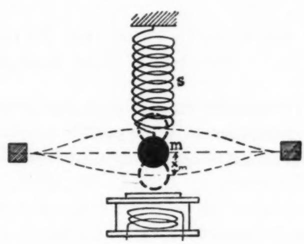

Fig. 7.

Diagrams Illustrating a Telephone-Receiver Diaphragm Vibratory System Considered as Replaced by its Equivalent Simple Vibrator.

which, moving with the velocity at the center, has the same kinetic energy as the whole diaphragm, with its actual distribution of mass and velocities.

r, the "motional resistance" of the diaphragm, in dynes per unit velocity (cm. per sec.), referred to the motion of the equivalent mass. This force, due to internal friction, eddy-current losses, and air propulsion on the diaphragm, is assumed to be directly proportional to the velocity, and dissipates the energy of motion.

s, the "stiffness constant" or the force per unit displacement (dynes per cm.), opposing the movement of the diaphragm, and referred to the equivalent mass at the center.

Thus in Fig. 6, the actual diaphragm, DD, clamped around the

2 C. G. S. Magnetic units are used in developing the theory, throughout this paper, with the distinguishing prefix ab- or abs-. The angle sign $\angle$ appended to the unit of an equation indicates that each term is a complex quantity or "plane-vector."

edge, is subject to distributed attracting electromagnetic and opposing resilient forces, with distributed mass or inertia, and distributed frictional resistance to motion. In Figure 7, the system is simplified to an equivalent central particle, of mass m, attracted electromagnetically, and having its motion opposed by a localized elastic tension of s dynes per cm. of displacement, together with a localized frictional opposing force[3] of r dynes per kine (dynes per cm per sec).

The justification for the above simplifying substitution, lies in the fact that the observed "motional-impedance circle," and its associated phenomena, are found to be in satisfactory accordance with the simplified theory.

Deductions from the Motional-Impedance Circle taken separately.

It was shown in the paper of 1912, that from an inspection of the motional circle, it is possible to obtain the following data:

(1) The resonant angular frequency of the diaphragm[4]

$$\omega_0 = \sqrt{\frac{s}{m}} \qquad \text{radians per second} \qquad (1)$$

This frequency is found at the maximum motional impedance, i. e. at OP, Figure 4, the diameter of the motional-impedance circle.

(2) The damping constant, or logn decrement per second of the diaphragm: $\Delta = \dfrac{r}{2m}$ numeric per second (2)

This damping constant is a function of the distribution of frequencies around the motional-impedance circle. It is equal to half the difference between the angular velocities at the quadrantal points QQ', Figure 4. In t seconds after the release of the diaphragm from a disturbed position, the amplitude of residual disturbance is $\epsilon^{-\Delta t}$ times the initial amplitude.

(3) At resonance, i. e., at the natural frequency of the diaphragm, the magnitude of the motional impedance, or the diameter of the motional impedance circle, is defined by

$$Z_m = \frac{A^2}{r} \qquad \text{absohms} \qquad (3)$$

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[3] The "kine" is a name for the C. G. S. unit of velocity, originally proposed by the B. A.

[4] See Appendix I.

The above data provide only three equations, (1), (2) and (3), whereas there are four unknown quantities to be determined; namely, A, m, r and s. A fourth independent equation, containing an additional experimental relation, is needed, in order to determine these four constants.

For supplying the missing fourth equation, it should be sufficient, theoretically, either to measure any one of the four constants directly, by an independent method; or to alter any one of the three constants m, r, and s, to a known extent, without changing the others, and then to repeat the circle diagram. From the difference between the circle diagrams, taken before and after the change, a fourth and independent equation should be forthcoming.

Object of the New Research here Reported.

The object of the research here described, was to ascertain how the missing fourth equation might best be obtained for evaluating A, m, r, and s. Several methods have been tried, and one, in particular, is recommended. Certainly by this preferred method, or perhaps by one of the other methods, these four constants can now be measured for any telephone receiver. Numerical values of these constants are tabulated for certain receivers.

Attention has also been given to the effects of variations in structure and environment, upon the motional-impedance circle diagram, and on the characteristic constants. Nearly one hundred circle diagrams, in all, on various receivers, have been made and examined during the course of the work.

Evaluation of the Telephone-Receiver Characteristic Constants A, m, r, and s.

The following methods have either been tried or suggested, at different times, for evaluating the characteristic constants of a telephone receiver.

(1) By the direct measurement of A.
(2) " " " " " s.
(3) " " " " " m.
(4) By loading the diaphragm with a known mass at the center.
(5) By varying the elastic constant s.
(6) By measuring the amplitudes of vibration over the air-gap.

Direct Measurement of A and *s*.

The direct measurement of both A and *s*, without reference to a circle diagram, was described by Abraham,[5] whose theory, for the particular case investigated, was limited to the use of these two constants only.

The direct measurement of A involves the determination of the pull on the diaphragm, excited by a known feeble continuous-current strength. It is doubtful whether satisfactory close approximations to the value of A can be made by continuous-current excitations; since alternating-current excitations are actually involved. With an exciting current strength of the same order of magnitude as is available with alternating-currents, the amount of diaphragm displacement is exceeding small, usually less than half of one micron;[6] whereas with a. c. excitation, at the resonant frequency, the displacements are commonly increased 30 times, and may be increased more than 50 times. The direct measurement of A is thus difficult, and subject to correction, for the change from continuous to alternating magnetic fluxes.

Direct Measurement of *s*.

Theoretically, the measurement of the elastic constant *s* of a diaphragm, in observing the central displacements produced by centrally impressed forces, appears both simple and promising. This measurement appears to have been first carried out by Abraham, in the publication above referred to; although the method is not disclosed. Attempts were made in the research here reported to measure *s* by means of the apparatus shown in Figure 8. The receiver T is mounted vertically, and the lever AB, fulcromed at F, and maintained in equilibrium by means of the counterpoise C, when the weight W is absent. A blunt point, on the lower side of this lever, is pressed down against the surface of the diaphragm, when the known weight W is applied. The force, in dynes, impressed on the center of the diaphragm is thus known. The point P of the micrometer screw-head M, comes into contact with the top of the lever AB, the moment of contact being determined electrically by the head telephones H, in the circuit I,

5 Bibliography, No. 6.

6 1 micron, commonly represented by $1\ \mu = 10^{-6}$ meter $= 10^{-3}$ mm.

of the voltaic cell E. Readings of the advance of the point P, could be made on the micrometer head to 10^{-3} inch ($25\,\mu$) and could be well estimated to 10^{-4} inch ($2.5\,\mu$). Impressed forces were used in succession up to about 100 grams wt. (10^5 dynes).

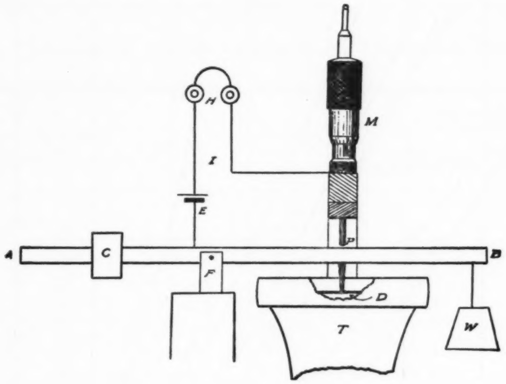

FIG. 8. — APPARATUS FOR DIAPHRAGM ELASTICITY MEASUREMENT.

Figure 9 shows the results obtained, with ordinates deflections in microns, and abscissas impressed central force in dynes. The different curves refer to diaphragms of different thickness. In every case the clamping circle of the diaphragm in the receiver had a diameter of 5.0 cms. The dotted curves show the deflections with the permanent magnet of the telephone removed. The full curves show the corresponding deflections with the permanent magnet present.

It will be seen that the deflections obtained with varying impressed forces, follow substantially straight-line laws, up to say 50 microns, a range of deflection considerably in excess of what the telephone diaphragm has ordinarily to undergo in practice. There is, however, a considerable difference between the deflection produced by a given impressed force when the permanent magnets were present, and when they were removed. This difference is accentuated in the cases of the thinnest diaphragms. The presence of the permanent magnet causes the diaphragm to be bowed from the plane of the clamping circle, and subsequent deflections, due to impressed forces, occur from this distorted position of equilibrium.

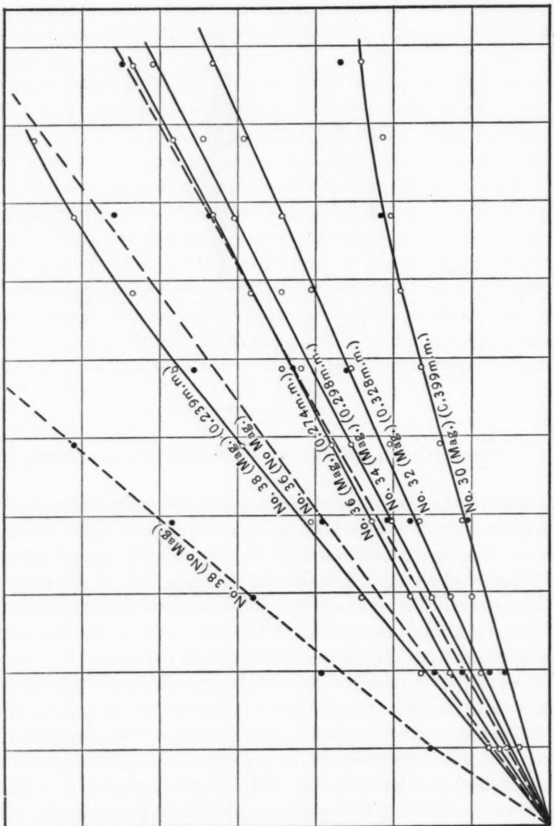

FIG. 9. — STATIC FORCE-DEFLECTION CURVES FOR DIFFERENT DIAPHRAGMS, WITH AND WITHOUT PERMANENT MAGNETS. HEAVY CIRCLES AND DOTTED LINES, WITHOUT MAGNETS.

The following Table gives the values of the elastic constant s of the diaphragms, as determined from the force-deflection curves in Figure 9. It should be noted that the sample diaphragm "japanned" was in

TABLE II.

Elastic Constants of Diaphragm from Static Deflection Measurements.

Sample No.	Thickness No.	Condition	Thickness over all	*s in megadynes per cm. deflection* With Permanent Magnet in Place	*s in megadynes per cm. deflection* With Permanent Magnet Removed
			mm.		
1	30	unjapanned	0.325	43.4	39.2
2	30	japanned	0.399	35.8	36.3
3	32	unjapanned	0.246	22.7	22.0
4	32	japanned	0 328	23.1	22.8
5	34	unjapanned	0.229	17.6	17.9
6	34	japanned	0.298	19.4	18.3
7	36	unjapanned	0.211	16.3	12.9
8	36	japanned	0.274	15.6	13.5
9	38	unjapanned	0.160	14.0	7.69
10	38	japanned	0.239	12.5	8.03

each case a different sample from the diaphragm of the same thickness number "unjapanned." Consequently, it is not safe to assume that the difference appearing in the Table between the values of s for a japanned and unjapanned diaphragm of the same thickness number, is due to the effect of japan. Such differences are more likely to be due to accidental variations in the thickness or quality of the two diaphragms. The japan coating had an average thickness of 0.075 mm. and does not seem to have appreciably affected the observed value of s. Below the thickness of plate No. 34, however, (0.23 mm.) the values of s, with the magnet in place, are markedly greater than with the magnet removed.

Judging from measurements of the effective value of s obtained by other methods, its direct determination by static deflection is unreliable, except as a rough approximation, for the purpose of arriving at the four essential constants of a telephone-receiver diaphragm, for the following reasons:

(1) In the case of static-deflection measurements, the forces are applied at the center of the diaphragm; whereas in a vibrating telephone diaphragm, the forces are applied over a considerable area of the surface, and with a different distribution of surface curvature, thus producing a different resultant effect.

(2) In the case of static-deflection measurements, the deflections are observed to increase with the time of application, especially with thin diaphragms. Consequently, the elastic constant, as deduced from static deflections, is likely to be less than would develop with rapid alternations of the impressed force.

Diaphragms numbers 2 and 6, in Table I, were submitted to a general analysis to be later described, by which the effective values of s were found to be respectively 77.6 and 36.7 megadynes per cm., values which are roughly twice as great as those given in the Table. It is believed, therefore, that the values of s obtained by static deflection, are much less than those which are effectively developed with impressed alternating forces.

Method of Direct Determination of m.

It is shown in Appendix II, that it is possible to determine the equivalent mass m, of a diaphragm, in terms of its total mass M, in active vibration, by exploring the vibration amplitude over the surface, when the diaphragm is actuated by a simple harmonic vibromotive force.[7] This method is theoretically capable of supplying the missing fourth equation, and, therefore, of determining, with the above mentioned data, all four constants, A, m, r, and s. This method has been developed, and actually carried out, in a number of instances for small diaphragms generally, and in two instances for the case of telephone-receiver diaphragms, operated electromagnetically.

While the method appears to be practically satisfactory, and its use throws much light upon the behavior of vibrating diaphragms; yet it is open to two objections, for the purpose here considered, namely:

(1) The technique of the exploration is both tedious and delicate.

(2) The telephone receiver must be screwed into a specially constructed amplitude-exploring apparatus, in which the upper surface of the diaphragm is left exposed; so that the telephone-receiver cap has to be removed, thereby altering, to some extent, the receiver characteristics.

This method is, therefore, not recommended for practical use.

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7 Bibliography, No. 17.

Method of loading the Diaphragm with a Central Mass.

It is shown in Appendix I, at formula (19), that the natural or resonant angular velocity of vibration of a diaphragm, considered as a simple vibrator, is expressed by the relation $\omega_0 = \sqrt{\frac{s}{m}}$. If, therefore, a known mass m_1 grams, is applied at the center of the diaphragm,[8] and the resonant angular velocity, under load, determined, we should expect it to be:

$$\omega_{01} = \sqrt{\frac{s}{m+m_1}} \quad \text{radians per sec.} \qquad (4)$$

provided that the vibratory behavior of the loaded diaphragm is the same as in the original unloaded condition, except in regard to resonant frequency. By combining (19) of App. I, with (4) immediately above, it should be possible to evaluate both s and m in terms of ω_0, ω_1 and m_1.

Effect of Adding a Mass m_1 to the Center of the Diaphragm.

The effect of adding a small metallic cylindrical mass m_1 grams, to the center of the diaphragm, so as to obtain equation (4), was investigated by Messrs. H. A. Affel and O. C. Hall, in a thesis for the Massachusetts Institute of Technology, on "Telephone-Receiver Characteristics," in 1914.

A standard bipolar Bell telephone receiver was used with the following dimensions:

Area of each pole in cm. × cm	1.4 × 0.225
Distance separating poles in cm.	0.85
External diameter of diaphragm in cm	5.40
Diameter of clamping circle in cm.	4.94
Thickness of diaphragm in cm	0.031
Total weight of diaphragm in gms.	4.181
Direct-current resistance of coils, ohms at 20°C	73.0

The electrical connections employed were the same as those in Fig. 1 of the 1912 paper, above referred to. The electrical measurements were all taken at constant voltage (0.42 volt r. m. s.) across the tele-

[8] Bibliography, No. 13.

phone-receiver terminals. With the diaphragm unloaded, the natural angular frequency was found to be $\omega_0 = 5645$ rad/sec. and the damping constant $\Delta = 211$ per second.

A small brass cylindrical mass of 0.98 gm. was then fastened, by shellac, to the center of the diaphragm. Its diameter was approximately 3 mm., and its height about 6 mm. The diaphragm thus loaded, was found to have a natural angular frequency of $\omega_{01} = 3980$ rad/sec. with a damping constant $\Delta_1 = 132$ per second.

Combining these two sets of observations, as in equations (4) and (19), we obtain, as the equivalent mass of the unloaded diaphragm,

$$m = \frac{\omega_{01}^2 m_1}{\omega_0^2 - \omega_{01}^2} = 0.968 \quad \text{gm.} \qquad (5)$$

Again, from (1) $s = \omega_0^2 m = 30.86 \times 10^6$ dynes/cm. (6)

or by (4) $s = \omega_{01}^2 (m + m_1) = 30.86 \times 10^6$ " " (7)

Similarly, from (2) $r = 2m\,\Delta = 409$ dynes/kine (8)

and (4) $r_1 = 2\,(m + m_1)\,\Delta_1 = 514$ " " (9)

The increase in the mechanical resistance of the loaded diaphragm may be accounted for either by increase in air resistance of the load, or in internal friction.

Finally from

$$(3) \quad A = \sqrt{Z_m r} = 6.114 \times 10^6 \quad \frac{\text{dynes}}{\text{absampere}} \qquad (10)$$

$$A_1 = \sqrt{Z_{m1} r_1} = \sqrt{103.2 \times 10^9 \times 514}$$

$$= 7.283 \times 10^6 \quad \frac{\text{dynes}}{\text{absampere}} \qquad (11)$$

The results of adding this load, and also two other successive loads, to the center of the diaphragm, are given in Table III. Here Column II gives the mass of the load in grams. Column III gives the measured angular velocity ω_0 of resonance, as obtained from the circle diagram in each case. It will be observed that with the heaviest load, the angular velocity was reduced nearly to one half of the initial angular velocity, unloaded. Column IV gives the motional-circle diameters in absohms. It will be seen that adding load to the diaphragm, first increased, and then reduced the dimensions of the motional-impedance circle. Column V records the depression angle of the diameter of the motional-impedance circle in each case. Column VI gives the damp-

ing constant $\Delta = r/(2m)$, or the logarithmic decrement per second, as determined from the distribution of angular frequencies around the circumference of the motional-impedance circle. Column VII gives the equivalent mass m of the diaphragm, as obtained in each case from formula (5). It should be constant according to theory, if formula (5) holds; i. e., if the relative distribution of vibration amplitudes over the surface of the diaphragm remains unchanged under different loads. The computed equivalent mass m of the unloaded diaphragm is seen to vary between 0.968 and 0.953 gm. Column VIII gives the inferred mechanical resistance r, in dynes per kine. It will be seen that r increased, when the load increased. This increase in mechanical resistance might be partly accounted for by the increased frictional

TABLE III.

Comparison of Computed Diaphragm Constants with and without Loads.

I	II	III	IV	V
Set No.	Added mass gm	Resonant Angular Frequency rad/sec.	Circle Diameter absohms	Depression angle degrees
	m	ω_0	Z_m	2β
1	0	5645	9.14×10^{10}	78.4°
2	0.980	3980	10.32×10^{10}	68.5°
3	1.588	3472	9.51×10^{10}	66.9°
4	2.447	2995	8.54×10^{10}	59.5°

VI	VII	VIII	IX	X
Damping Constant per sec.	Equivalent mass. gms.	Mech. Resist. dynes/kine	Stiffness Constant mega-dynes/cm.	Force-factor mega-dynes/absamp.
	m	r	s	A
211		409	30.86	6.114
132	0.968	514	30.86	7.283
119	0.953	600	30.38	7.554
100	0.959	681	30.55	7.627

resistance of the load in air. Column IX gives the computed stiffness-constant s, as computed from formulas (1) and (4). Theoretically, this stiffness coefficient should remain unchanged throughout. According to the Table, it varied between 30.86 and 30.38 megadynes per cm. Column X gives the force-factor A according to formulas (10) and (11). It will be seen that as the load was increased, and the natural frequency reduced, this force-factor increased from 6.114 to 7.627 megadynes per absampere. This increase may be explained by the change in penetration of the alternating magnetic flux into the substance of the core and poles, as the frequency is changed.[9] This change in the force factor A tends to distort the motional-impedance circle to some extent; but since, in the majority of instances, most of the circumference is covered within the range of 100 cycles per second, the actual distortion of the diagram on this account is seldom serious.

The method of computing the telephone-receiver characteristic constants by means of mass added to the diaphragm, as outlined above, involves the assumption expressed in (5) that the equivalent mass m of the diaphragm is the same in both the loaded and unloaded states. The theory of the equivalent mass of a diaphragm is outlined in Appendix II. It will be seen that the equivalent mass is always equal to the actual mass of the vibrating area, or area within the clamping boundary of the diaphragm, multiplied by a numerical coefficient, which may be called the "equivalent-mass factor," whose value depends upon the distribution of vibration amplitudes over the entire vibrating surface. If the entire active surface of the diaphragm could vibrate with the same amplitude as actually exists at the center, the equivalent-mass factor would be unity. Since the amplitude must diminish towards the boundary, the factor is always less than unity, and may lie, say between the limits 0.15 and 0.50, ordinarily between 0.2 and 0.3. When a mass m_1, is added to the center of a diaphragm, it is apt to disturb the distribution of amplitude over the vibrating surface, and thus change the equivalent mass m of the diaphragm, considered alone; so that the total equivalent mass of the loaded diaphragm is no longer $m + m_1$. The amount of the disturbance in m created by the load will depend upon various factors, such as the geometry of the diaphragm, the position of the electromagnetic poles, and the magnitude of the load m_1; so that a load which might appreciably disturb the equivalent mass of one receiver diaphragm, might not appreciably affect the equivalent mass of another.[10] In the case

9 The theory of this effect has been developed by Dr. R. L. Jones, and is expected to be published shortly.
10 Bibliography, No. 17.

presented in Table III, it will be seen that the effect of loading the diaphragm, with masses up to nearly 2.5 gms., did not seriously affect the equivalent mass m of the diaphragm, as deduced from the observations recorded in Column VII. On the other hand, similar loads, applied to other diaphragms, have been followed by changes in equivalent mass, with corresponding vitiations of results.[11]

It has also been found, in certain cases, that while the addition of a load to the center of the diaphragm changed the equivalent mass of the diaphragm considered by itself, the addition of further loads, seemed to leave the equivalent mass of the diaphragm with the first load practically unchanged. Consequently, if say three different increasing loads are added in succession, the solutions of simultaneous equations, using the unloaded and any loaded case, will not be consistent; whereas solutions of simultaneous equations using different loaded conditions may be consistent.

The following case may illustrate the preceding remark. A telephone receiver with an active diaphragm weight of 4.94 gm. and 0.399 mm. thick, over japan, was tested by the motional-impedance circle, first unloaded, and then with three successively increasing brass loads of 0.615, 0.978, and 2.998 gms. The resonant angular velocities in these four cases were respectively (1) 7570, (2) 6568, (3) 5990, and (4) 4289 radians per second. From these data, we obtain the following results, using formula (5), entered with various pairs of observations:—

TABLE IV.

Observation Pair	Equivalent mass of unloaded diaphragm	Equivalent-mass factor
Unloaded and load, (1) and (2)	1.88 gm.	0.380
" " " (1) " (3)	1.64	0.332
" " " (1) " (4)	1.42	0.288
Two loads, (2) " (3)	1.18	0.239
" " (2) " (4)	1.16	0.235
" " (3) " (4)	1.15	0.233

11 Dr. Jones reports having obtained satisfactory results with the loading method, in various cases when the total added mass was not greater than 0.3 gm.

It will be seen that the equivalent mass of the diaphragm alone, as deduced from observations unloaded and with some one load, vary from 1.88 to 1.42 gms., whereas the equivalent mass, as deduced from observations with different loads, only varies between 1.15 and 1.18 gms.

Owing to the possibility of introducing a change in the equivalent mass, or other constants of the unloaded diaphragm by the addition of a load, the method of determining m by adding loads should be considered as open to criticism, unless sufficiently checked.

Method of Maximum Resonant Amplitude Measurement over an Air-Gap.

It is shown at the end of Appendix I, that if the maximum cyclic amplitude of vibration is known at a point on the diaphragm over an air-gap, and also the maximum cyclic alternating-current strength producing this vibration, the missing fourth relation is arrived at; so that with the aid of the motional-impedance circle, all four characteristic constants are determinable. This method should have the advantages, that it need not interfere appreciably with either the structure, or the normal mode of operation, of the telephone receiver. It also has the advantage that the amplitude of vibration, which is usually so small for the feeble currents of normal operating strength, becomes very appreciable at the resonant frequency, and is ordinarily about 5 microns per milliampere of testing current. Strictly speaking, the amplitude should be measured over the air-gap, whose cyclic variation induces the motional emf.; but ordinarily, it suffices to measure the amplitude at or near to the center of the diaphragm. The method was evolved from the experimental exploration of vibration in diaphragms,[12] and the apparatus used is a simplified form of such an explorer, for measurements at the center, instead of at any or all points on the surface.

Amplitude Measurer.

The amplitude measurer is illustrated in Figs. 10 and 11. The brass clamping frame F can be attached to the cap of any ordinary receiver, and serves to support the working parts of the instrument.

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[12] Bibliography, No. 17.

A small triangular mirror M of very thin silvered glass, and say of 1 mm. in length of each edge, is fastened to a little phosphor-bronze strip, about 4 mm. long, stretched between two pointed metallic abutments. The strip is so fastened that one point of the mirror is pressed elastically into contact with the diaphragm D, to the center of which it is presented. The purpose of the mirror is to reflect a beam of light, from a powerful lamp, on to a graduated scale; so that vibrations of the diaphragm will cause the reflected beam to extend into a band of light on the scale. Since the effective radius of lever arm in the mirror is about 1/3rd mm., the amplitude of motion of the diaphragm can readily be magnified, say 8000 times, on a scale

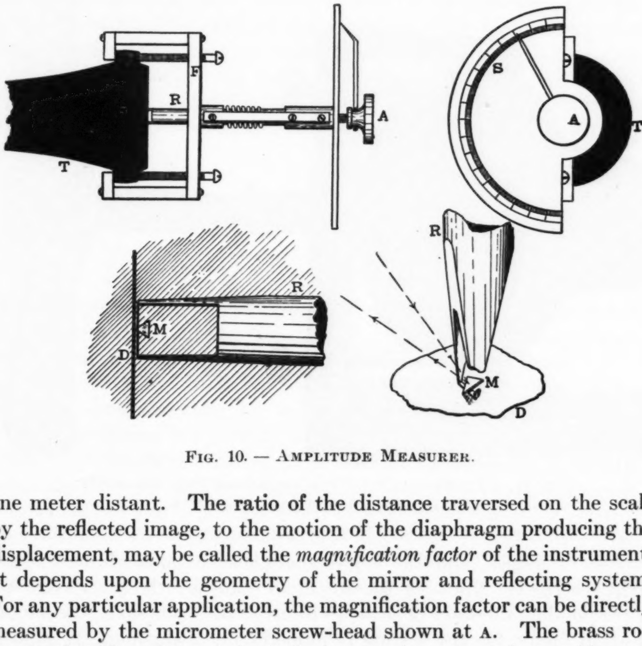

FIG. 10. — AMPLITUDE MEASURER.

one meter distant. The ratio of the distance traversed on the scale by the reflected image, to the motion of the diaphragm producing the displacement, may be called the *magnification factor* of the instrument. It depends upon the geometry of the mirror and reflecting system. For any particular application, the magnification factor can be directly measured by the micrometer screw-head shown at A. The brass rod R, carrying the mirror on its forked extremity, moves in a guide, and its movement parallel to the axis of the telephone receiver is controlled by the screw head A. One turn of the screw head A advances the mirror M towards the diaphragm through a distance of one thread,

against the tension of the spiral spring shown. In the particular screw used, there were 24 threads to the inch, or 1 turn = 1.06 mm. The semicircular divided scale s enables the advance of the screw to be measured to $\frac{1}{2}$ degree, and estimated to $\frac{1}{10}$ degree; so that the advance could be measured to 1.5 μ, and estimated to $\frac{1}{3}$ μ. With the mirror in working contact on the diaphragm at rest, the position of the reflected spot of light is observed on the scale. The screw head A is now turned steadily, in one and the same direction, through several

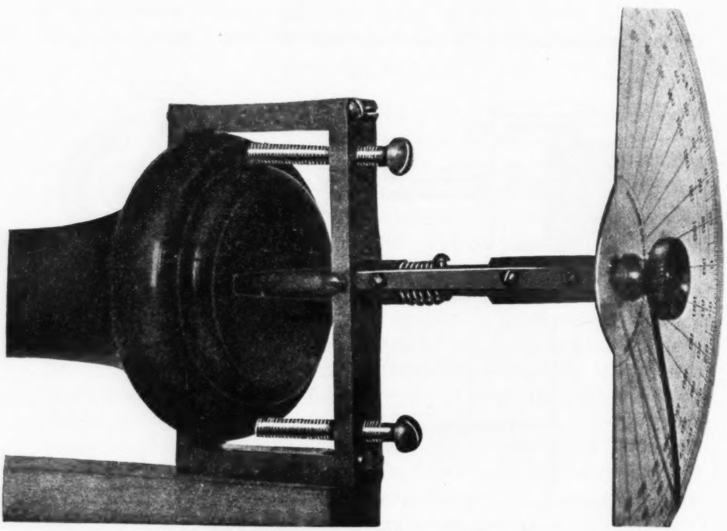

FIG. 11.—PHOTOGRAPHIC VIEW OF AMPLITUDE MEASURER APPLIED TO A TELEPHONE RECEIVER.

successive angles, and the corresponding excursions of the spot of light recorded. Knowing the advances made by the screw, and the corresponding numbers of scale divisions in the excursions, the magnification factor for the setting is immediately deducible. A calibration curve for magnification factor of spot deflection, versus-screw head angular motion, usually follows a straight line, over say 60 microns of diaphragm amplitude; provided that the range of scale deflection does not commence with the mirror almost breaking contact

with the diaphragm. It is assumed in this calibration, that the diaphragm remains undeflected from its position of rest through the working range of luminous-spot deflection. With ordinary telephone receiver diaphragms, it is found that the small pressures exerted by the mirror produce no appreciable deflections, as these pressures are less than 100 dynes, and the curves in Figure 9, show that such pressures have inappreciable deflecting influences.

In order that the exploring mirror M shall not break out of contact with the vibrating surface of the diaphragm, to which it is applied, it is necessary and sufficient that (1) statically, when the diaphragm is at its maximum downward displacement, the point of the mirror shall at least be in contact therewith, and (2) dynamically, that the natural vibration frequency of the mirror shall be higher than that of the diaphragm's vibrations under test, at resonance. This means that it must ordinarily have a natural frequency of from 1000 to 1500 cycles per second. No difficulty has been found in accomplishing this result. The phosphor-bronze strip does not need to be stretched very tightly.

Technique of Procedure for Determining Constants by Method of Amplitude.

The full procedure for determining the constants A, m, r and s of a telephone receiver, using the method of amplitude measurement, which the authors have found to be advantageous, is as follows:

Connect the receiver to be tested in a Rayleigh bridge, Figure 12, for measuring simultaneously its apparent resistance and inductance, at various impressed frequencies, with a known r. m. s. testing-current strength. A Vreeland oscillator [13] is used as the source of sinusoidal alternating currents, and by means of the resistance R, the voltage at the terminals A^1 and D is maintained at a constant value by electrostatic voltmeter VM. An anti-inductive resistance R^1, sufficiently large to keep the current in the bridge constant [14] is inserted between A and A^1 as shown. The two arms of the bridge AB and AC have equal

13 Bibliography, No. 9.

14 The method of testing at constant current is theoretically an improvement over that of testing at constant voltage, and is due to the work of the Western Electric Co.'s Engineering Department. The difference between results at constant bridge voltage and constant bridge current, are, however, ordinarily trivial.

resistances, wound anti-inductively. The arm CD contains adjustably variable resistance and inductance, for balancing the tested receiver in BD. A pair of head telephones in the bridge wire BC, enables the balance to be obtained conveniently, at any frequency between 400 and 2500 cycles per second, the range employed. It has also been found convenient to use in each of the bridge arms AB and AC, a resistance of 200 ohms, when testing ordinary receivers of about 75 ohms d. c. resistance, and for the purpose of the amplitude measurements, a testing current of 2 milliamperes r. m. s. in the receiver; or 4 milliamperes in the bridge through R^1, with say 10 volts across A^1D, which calls for about 2250 ohms in R^1. A series of resistance and inductance measurements of the undamped receiver T, are then made over the

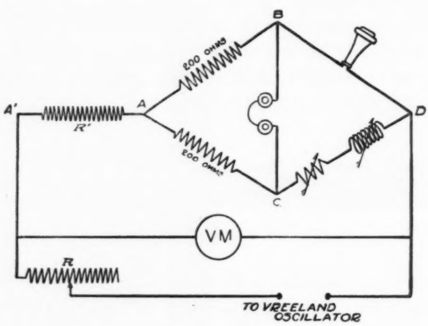

FIG. 12. — INDUCTANCE BRIDGE CONNECTIONS FOR TESTING TELEPHONES.

whole range of frequency to be covered, the frequency supplied by the Vreeland oscillator being increased by convenient successive steps, through adjustment of the capacitance in the Vreeland primary circuit. Near the resonant frequency of the receiver, these steps have to be increased in number, and diminished in size, in order to follow the rapid variations in the telephone impedance, as the frequency passes through resonance. The free impedance of the telephone thus becomes known over the range of impressed frequency.

The amplitude measurer is then applied to the telephone as already described, and the maximum double amplitude, on its scale, of the band of light formed by the oscillation of the reflected beam is adjusted for in, frequency. It is also found very convenient to record this amplitude over the entire range of frequency measurements. Half

the double maximum amplitude of the luminous band, divided by the magnification factor, as found for the setting, should give the true maximum cyclic amplitude x_m of the diaphragm from its normal position of rest.

The amplitude measurer is then removed from the receiver T, and a damping device is applied to the diaphragm. A corresponding series of simultaneous inductance and resistance measurements is then made over the same range of frequency as before, with the telephone diaphragm damped, so as to suppress its vibrations. The application of a finger to the center of the diaphragm will serve to damp out the vibrations; but this is not a satisfactory damping method; because,

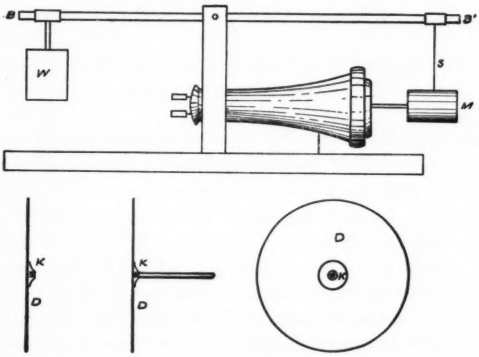

FIG. 13. — METHOD OF DAMPING TELEPHONE DIAPHRAGM

first, the pressure is not uniform during the period of testing, and secondly; because any pressure on the diaphragm, by diminishing the air-gap, alters, to some extent, the magnetic characteristics of the instrument. A convenient form of damping device is indicated in Figure 13. It virtually applies a relatively large mass to the diaphragm, without imposing any appreciable mechanical pressure; so that inertia is depended upon for extinguishing the vibration. The receiver is securely supported in a horizontal position. A small threaded metallic disk K, is attached, by means of shellac, to the center of the diaphragm. Thick shellac varnish appears to be a convenient material for cementing a load to the diaphragm. The cylindrical metallic mass M (4 cm. long and 2 cms. in diameter) is then connected mechani-

cally with the disk K by means of a screw rod. The weight of the cylinder M is carried by the flexible metallic strip S, and counterbalanced on the beam BB[1] by the weight W. When carefully applied, the receiver diaphragm D at rest, remains damped without being stressed.

The vector differences between the free and damped impedances,[15] over the range of impressed frequency, will enable the motional-impedance circle to be drawn.

From the data thus obtained, all four constants of the instrument may be computed as indicated in Appendix I. The circle diagram supplies ω_0, Δ, and Z_m. The amplitude measurer gives x_m. The conditions of the Rayleigh bridge, and its voltmeter, give I_m.

Data Obtained on Particular Receivers by Means of Amplitude Measurements.

The following are the results of tests on four telephone receivers, marked A, B, C and D respectively, using the amplitude method, in conjunction with the motional-impedance circle diagram. Two of these, A and B, were tested in a vibration explorer [16] previously referred to, and the other two — C and D,— were tested with the amplitude measurer above described. Although the tests with the vibration explorer interfered with the normal structure of the instrument, by reason of the removal of the receiver cap, and are therefore not completely representative of the normal operating conditions; yet it was deemed important to secure, from the vibration explorer, a check on the amplitude method, through an independent determination of the equivalent mass.

The Table is divided into three parts. The first gives the Data obtained from the dimensions of the poles and of the particular diaphragm used; also the d. c. resistance of the winding at 20°C, by Wheatstone Bridge. The telephone receiver D, was a "1000-ohm" instrument. The others were of the usual "75-ohm" type.

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15 It has been suggested by the Engineering Department of the Western Electric Co., that if two receivers are available, with sufficiently similar constants, one (free) may be inserted in the arm BD, and the other (damped) in the arm CD. A single series of inductance and resistance measurements, over the range of impressed frequency, will then be sufficient to determine the motional impedance directly.

16 Bibliography, No. 17.

TABLE V.

DATA SECURED ON PARTICULAR RECEIVERS.

Receiver Number	A	B	C	D
Type	Bell Bip.	Bell Bip.	Bell Bip.	Watch Case Bip
Data				
Area of each *Pole* cm. × cm.	1.14 × 0.199	1.40 × 0.225	1.14 × 0.199	1.15 × 0.18
Distance separating poles cm.	0.686	0.85	0.686	1.10
External diam. of diaph. cm.	5.52	5.40	5.52	5.57
Diameter of clamping circle cm.	5.00	4.94	5.00	4.95
Thickness of diaphragm over japan cm.	0.0399	0.031	0.031	0.0244
Wt. of diaphragm gm.	5.979	4.181	4.397	3.365
Direct current *Resistance* of coils, ohms at 20° C.	87.1	73.0	86.7	1079
Test Data				
Temperature of Test Deg. C.	26.7°	27.8°	20.°	20°
Current through Rec. absamps. *r.m.s.* I	0.000 202	0.000 200	0.000 204	0.000 116
Resonant Freq. of Rec. cyc./sec. f_0	993	1020.4	1015	898.5
Resonant Ang. Vel. rad /sec. ω_0	6240	6412	6378	5646
Motional Impedance Circle diameter absohms. Z_m	80.2 × 10^9	70 × 10^9	140 × 10^9	367 × 10^9
Max. Amplitude of Resonance cm. × 10^{-4} (Microns). x_m	7.53	7.19	10.35	6.64
Velocity of Diaph. at Resonance cm./sec. (max. cyclic). $\dot{x}_m$	4.70	4.62	6.6	3.75
Decrement per sec. Δ (Mean)	61.2	236	149	356

TABLE V (*Continued*).

Receiver Number	A	B	C	D
Type	Bell Bip.	Bell Bip.	Bell Bip.	Watch Case Bip.
Calculated Data				
Equivalent *Mass* of Diaph. m, (gms.)	*2.41*	*0.557*	*0.902*	*0.986*
Equivalent *Mass Factor*	0.49	0.17	0.245	0.387
Equivalent Mass Factor by Exploration Method	0.53	0.18		
Equivalent *Elasticity*, s, dynes per cm.	94.0×10^6	22.9×10^6	36.7×10^6	31.44×10^6
Equivalent Diaph. *Resistance* r, dynes per kine	*295*	*262.3*	*268*	*702*
Force Factor, A, dynes per absampere	4.87×10^6	4.28×10^6	6.12×10^6	16.05×10^6
Mean Angle of Lag β degrees	29.7	37.3	25.3	37.0

The second part of the Table gives the results obtained from the motional-impedance circle, and from the amplitude measurer. The circle diagram for the case of receiver B, is given in Figure 14. The outer circle is the motional-impedance circle, plotted to a scale of ohms, from observations with the Rayleigh bridge (Fig. 12). The inner circle, is a circle of maximum cyclic vibration velocities, at the center of the diaphragm, as deduced from vibration amplitudes observed with the vibration explorer, plotted vectorially from the origin O. It is shown in Appendix I (40) that the motional impedance, when the sinusoidal testing current is maintained constant, becomes the product of the vibrational velocity $\dot{x}$ and a constant. The inner circle, thus conforming satisfactorily to the impedance circle, supplies a check upon the theory of the case. Theoretically, the mean vibration amplitude should be measured over the air-gap, instead of at the center of the diaphragm; but the difference is probably not material. It will be noticed that the resonant frequency f_0, on the diameter of the impedance circle, is 1020.4 ∾, with a corresponding resonant angular velocity ω_0, of 6412 radians per second. The frequencies at the

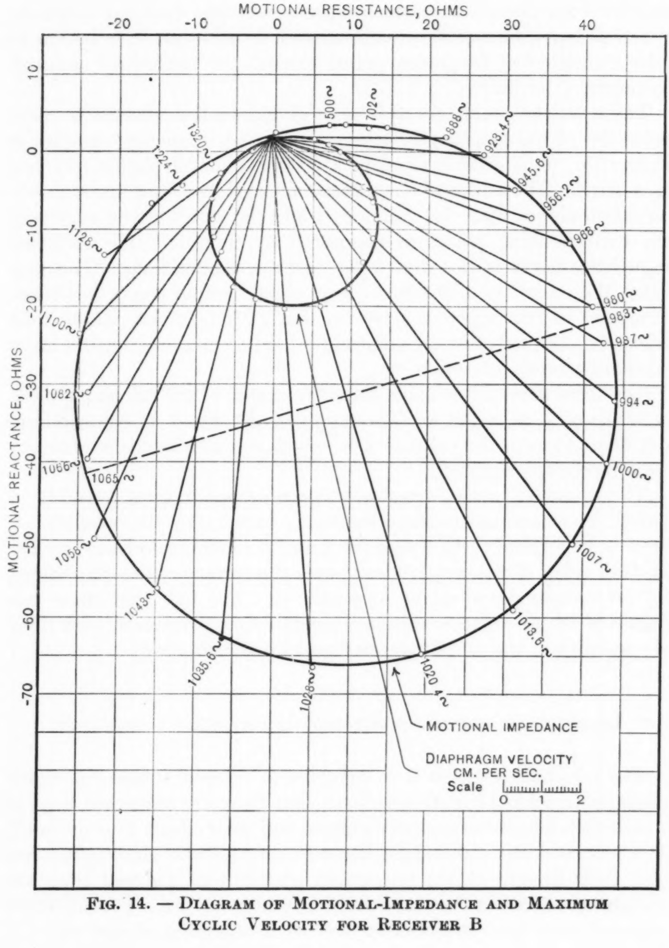

Fig. 14. — Diagram of Motional-Impedance and Maximum Cyclic Velocity for Receiver B

quadrantal points of the circle are 983 and 1065 ∾ respectively, or in angular velocity 6176 and 6692 radians per second. Half the difference between these is 258, which represents Δ, the damping constant. When Δ is required with greater precision, however, it is better to take a number of frequency points around the circle into account, by means of formulas (20) to (23).

The resonance curve for this case, giving central maximum cyclic velocities of the diaphragm, against impressed frequency, appears in Figure 15. It is a fairly representative curve for telephonic receivers as a class. The curve of diaphragm velocity is drawn through observation-points with the explorer, while the small circles represent the corresponding points as computed by formula. The motional impedance curve is also drawn through the observations. Theoretically, the ordinates of the two curves should retain a constant ratio. It should be remembered, however, that at frequencies remote from resonance, not only are the amplitudes hard to measure, but the force constant A needs correction.

The third part of the Table gives the results of the calculated data, in terms of A, *m*, *r* and *s*. The numbers of turns N, in the windings not being known, the value of A/N, which is a characteristic quantity for a receiver, is omitted.

The equivalent mass factor *m*/M was obtained in tests A and B by two different and independent methods; namely, by the computation of *m* as in Formula (49) Appendix I, and through exploration, by Dr. H. O. Taylor, of the amplitudes [17] over the surface of the diaphragm, in the manner described in Appendix II. The fact that these two values of the mass-factor compare favorably, constitutes a check upon the validity of the amplitude method of determination.

INFLUENCES WHICH AFFECT THE INSTRUMENT CONSTANTS.

The telephone receiver is so sensitive to external influences, which effect the motional-impedance circle, that there are numerous ways in which such influences might be exerted and their effects thus revealed. It is proposed here to consider, however, only a few of these influences, and their general effects on certain receivers as deduced from the changes produced in their motional-impedance circles. The influences selected were (1), variations in the screw-clamping of the cap, (2),

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17 Bibliography, No. 17.

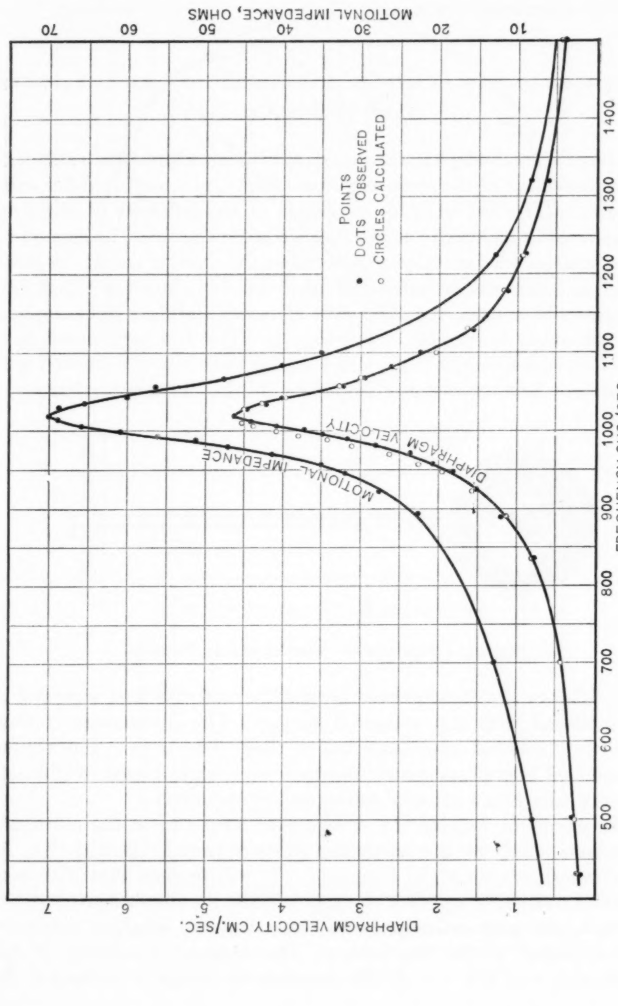

FIG. 15. — RESONANCE CURVES OF DIAPHRAGM VELOCITY AND MOTIONAL IMPEDANCE, RECEIVER B

variations in temperature, (3), variations in air chamber between cap and diaphragm, (4), variations in atmospheric pressure, (5), variations in added mechanical resistance.

Variations in the Clamping Adjustment of the Cap under Varying Torque.

Tests of several telephone receivers, made before and after a removal and replacement of the screw cap, were found, at times, to differ considerably. This led to an investigation of the influence of screwing on the receiver cap with varying degrees of tightness. The caps were of the same molded composite material as the receiver cases. A lever clamping device was designed and constructed as shown in Figure 16. It consists of a brass rod AB, with a known sliding weight applied at a measured horizontal radius arm R. The rod terminates in a brass fork containing notches, which engage with pins P P screwed into the cover. The receiver is clamped by its shell in a horizontal posi-

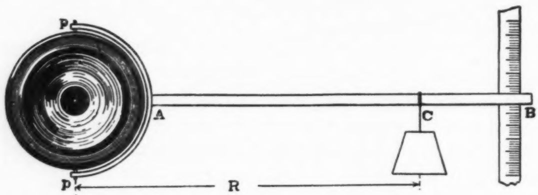

Fig. 16. — Method of Applying Cap Torque.

tion. The cap is then screwed on slackly, and the final screwing is accomplished with the measured torque. The instrument is then tested for motional impedance under these conditions. The torque is expressed in gram-perpendicular-meters; i. e., in grams weight acting vertically at a horizontal radius arm of one meter.

The effects of varying the screwing-on torque upon the motional-impedance circle of the particular receiver tested, (B with No. 36 diaphragm), are shown in Figure 17. It will be seen that with zero imposed torque; i. e., with the cap laid on the diaphragm, but not screwed, the motional-impedance circle has the smallest diameter, and is nested within the others. The resonant frequency of the diaphragm was 859 ∾. As the screwing-on torque is increased, the

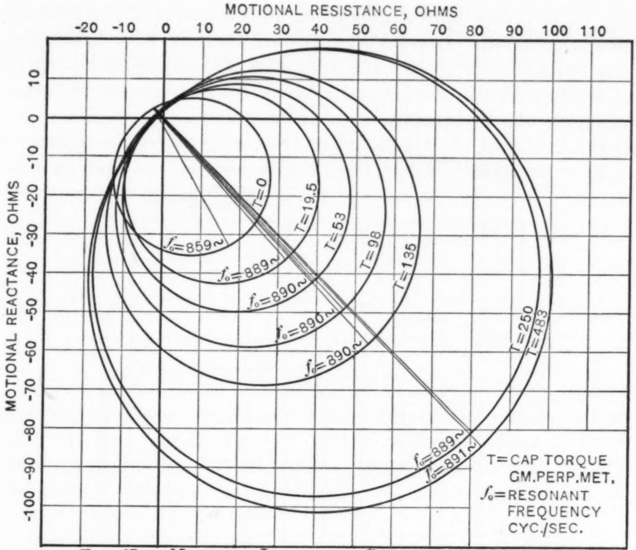

FIG. 17. — MOTIONAL-IMPEDANCE CIRCLES WITH DIFFERENT CAP ADJUSTMENTS.

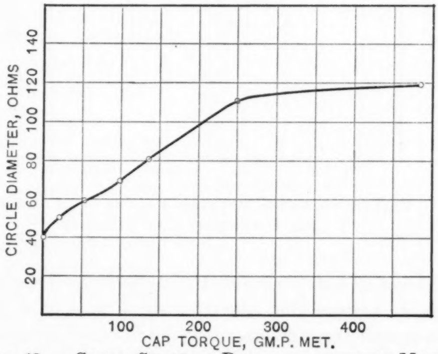

FIG. 18. — CURVE SHOWING RELATION BETWEEN MOTIONAL-IMPEDANCE CIRCLE DIAMETER AND CAP TORQUE.

diameter of the motional-impedance circle increases, as indicated in Figure 18, in nearly simple proportion, until a torque of 250 gm-perp-meters is attained. Beyond this torque, there is very little effect on the circle diameter. After a torque of 20 gm-p-m had been applied, no appreciable effect was discovered on the resonant frequency of this particular instrument.

At the time that the above torque tests were taken, the method of amplitude measurement had not been developed; so that an exact

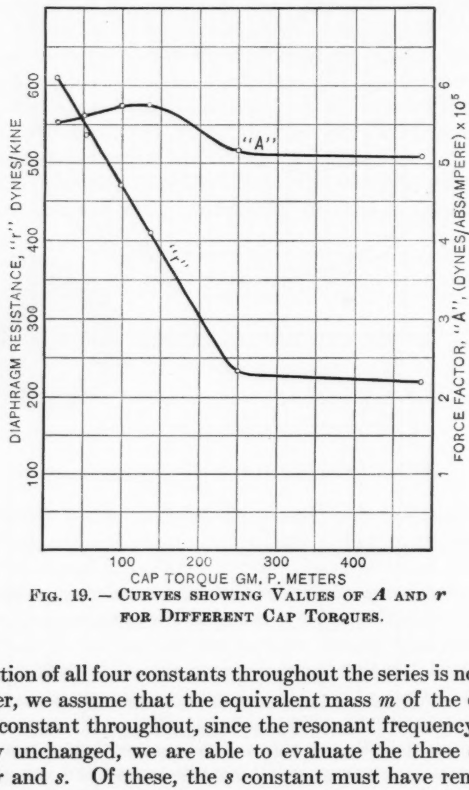

Fig. 19. — Curves showing Values of A and r for Different Cap Torques.

determination of all four constants throughout the series is not possible. If, however, we assume that the equivalent mass m of the diaphragm remained constant throughout, since the resonant frequency remained practically unchanged, we are able to evaluate the three other constants A, r and s. Of these, the s constant must have remained unchanged, and the only variations would be those of A and r, which are plotted in Figure 19 as ordinates against torques as abscissas. It will

be seen that the value of A remained but little changed throughout the series, diminishing from 0.575 to 0.51 megadyne per absampere. The mechanical resistance r of the diaphragm changes, however, largely, diminishing from about 600 to 200 dynes per kine.

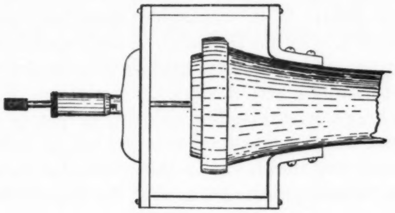

FIG. 20. — METHOD OF AIR GAP MEASUREMENT.

At first sight, it seems difficult to explain why the change in imposed screwing-on torque should affect the mechanical resistance so markedly. It was found, however, that a noticeable effect of screwing on the cap tightly, was to increase the air-gap between diaphragm and poles. This is a reasonable effect, if it is remembered that the magnet poles bow the diaphragm down towards them, when the diaphragm is laid on the clamping ring, and that the application of cap pressure, under the influence of screwing torque, to the upper clamping ring, tends to lessen this bowing.

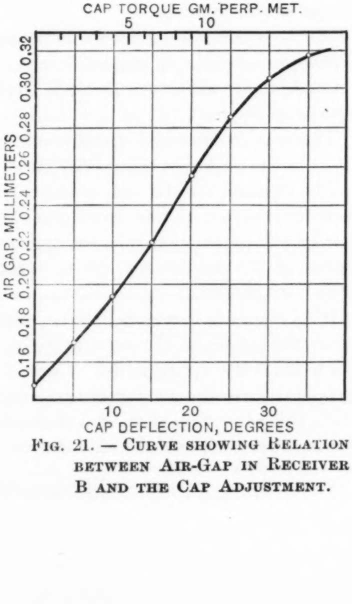

FIG. 21. — CURVE SHOWING RELATION BETWEEN AIR-GAP IN RECEIVER B AND THE CAP ADJUSTMENT.

The increase in air-gap accompanying increase in torque was measured by the device shown in Figure 20, where the micrometer-head depth gauge is applied to the upper surface of the diaphragm, from a temporary brass frame attached to the shell. Figure 21 shows the magnitudes of the deduced air-gaps, as ordinates, against

degrees of twist of the cap as abscissas, an approximate scale of corresponding imposed torque being added.

It is known that a considerable proportion of the mechanical resistance *r* offered by a receiver diaphragm, is due to the braking effect of eddy currents set up in the diaphragm, during its vibration in a strong magnetic field. It is reasonable, therefore, to suppose that the diminution in *r*, which was found to accompany the increase in screwing-on torque, and in air-gap, may have been due to a reduction in eddy-current damping action, since the variations of magnetic field within the diaphragm would be diminished. It may be safely inferred that the adjustment of a telephone-receiver cap is able to affect the receiver constants considerably. In this particular receiver, the most sensitive and satisfactory setting was with the cap screwed on tightly.

Effect of Variations in Temperature.

In order to ascertain the influence of temperature upon the characteristics of a receiver, a large electric oven [18] was used, in which the receiver was placed, at a conveniently controlled temperature. A number of motional-impedance circles were observed, at oven temperatures from 16° C. to 50° C., all other conditions being maintained constant. It was found that two effects were produced with rise of temperature; namely:

(1) A reduction in resonant frequency, amounting to about 2.5 cycles per second, per degree C. rise.

(2) A slight reduction in circle diameter, which, however, was not always noticed.

The results obtained are given in the following Table. They are given in the sequence of observation.

The reasons for the above indicated effects of temperature on the receiver characteristics have not been analysed. They might be attributed to temperature changes in the mechanical elasticity constants of the diaphragm; or to expansional effects in the structure; or to both causes.

It is evident, therefore, that, judging from this particular instrument, the influence of temperature on a telephone receiver's characteristics are very appreciable. Care should be taken to maintain the temperature of the instrument constant during any set of observations. With this object in view, it was customary to conduct the tests with the instrument inside the closed oven, and with the heat shut off.

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[18] Bibliography, No. 18.

TABLE VI.

TEMPERATURE EFFECT ON RECEIVER CHARACTERISTICS.

	Series A		Series B			Series C		
Condition of Cap Adjustment	Tight	Tight	Loose	Loose	Loose	Tight	Tight	Tight
Temperature deg. C.	19 3°	47.0°	16.0°	46.0°	31.5°	51.0°	36.0°	22.0°
Resonant Frequency cyc/sec. f_0	886.7	834.9	827	742	788	817.3	852.5	867.5
Resonant Ang. Vel. rad/sec. ω_0	5575	5248	5200	4666	4956	5140	5360	5450
Motional Impedance circle diam. ohms Z_m	88.2	72.0	52.0	48.0	45.5	70.5	70.0	71.0

EFFECT OF VARIATIONS IN AIR-CHAMBER BETWEEN CAP AND DIAPHRAGM.

In order to ascertain the influence of the air-chamber over the diaphragm of the ordinary receiver, a special cap was used, in which this air-chamber could be varied, by altering the position of a friction-tight cylindrical plug, of the same diameter as the clamping circle. Motional-impedance circles were observed under these different conditions, with the results given in the following Table:—

TABLE VII.

RESULTS OF MOTIONAL-IMPEDANCE CIRCLES WITH CHANGES IN CAP AIR-CHAMBER.

Air-Chamber Thickness	Circle-Diameter ohms	Resonant Frequency cycles per sec.
Normal, 0.5 mm.	139	872
Large, 5 mm.	150	892
Infinite, Plug Removed	155	927

The effect of the air chamber, as compared with an open diaphragm, was to lower the resonant frequency slightly, and to reduce the circle diameter.

In order to ascertain the influence of the ordinary atmospheric pressure upon the characteristics of the diaphragm, a bipolar receiver was suspended in the glass bell-jar of an air-pump, the two wires to the receiver being carried through a seal at the top of the jar. One motional-impedance circle was obtained with full atmospheric pressure, and another after the pressure had been reduced to about 1 cm. of mercury. The two circles are shown in Figure 22. It will be seen that the removal of atmospheric pressure decreased the resonant frequency and enlarged the circle diameter. The results are given in the following Table:

TABLE VIII.

THE EFFECT OF THE ATMOSPHERE ON RECEIVER CHARACTERISTICS.
Receiver C.

Condition of Receiver	In Vacuum	In Air
Temperature, deg. Cent.	18.5°	18.5°
Current, absamperes. *r.m.s.* I	0.000 05	0.000 05
Resonant Frequency, cyc/sec. f_0	956	1020
Resonant Ang. Velocity, rad/sec. ω_0	6007	6409
Impedance Circle Diam. ohms Z_m	177	116
Decrement per sec. Δ	120	176
Equivalent mass gm. m	0.902	0.902
Equivalent Elasticity, dynes/cm. s	32.5×10^6	37.0×10^6
Equivalent Resistance, dynes/kine. r	216.5	317.5
Resistance due to Air, dynes/kine = 31.8% of total resistance	101.	

Assuming that the equivalent mass m remained unchanged, the equivalent resistance r diminished in vacuo by about 30 per cent. The power of the receiver as a sound-producing device is theoretically limited to $(\dot{x})^2r$ or $101(\dot{x})^2$ abwatts. Owing to the fact that the underside of the diaphragm is cut off from free access to the air, only part of this power can be actually utilised for sound production in air.

At certain frequencies near resonance, the Rayleigh bridge balance becomes very sensitive to variations in atmospheric pressure. From an examination of the balance at these frequencies, it was easy to ascertain whether air was leaking into the bell jar. It seems likely,

therefore, that sudden variations of barometric pressure, within the normal range, might produce perceptible changes in the free impedance, near resonance.

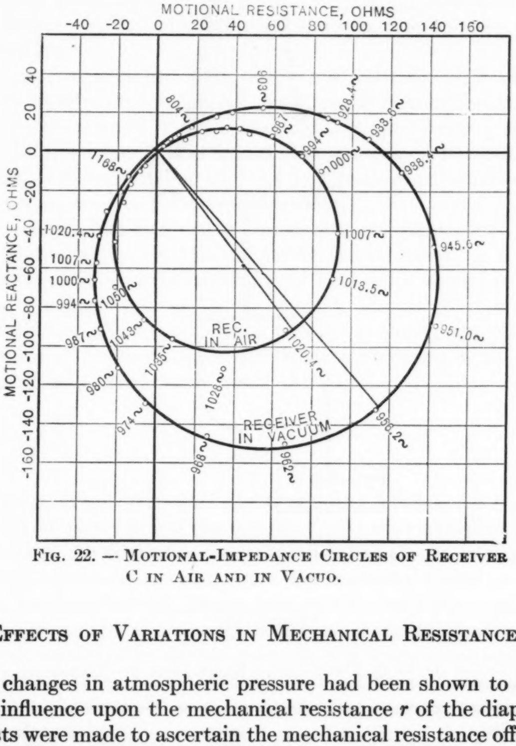


FIG. 22. — MOTIONAL-IMPEDANCE CIRCLES OF RECEIVER C IN AIR AND IN VACUO.

EFFECTS OF VARIATIONS IN MECHANICAL RESISTANCE.

Since changes in atmospheric pressure had been shown to effect a marked influence upon the mechanical resistance *r* of the diaphragm, some tests were made to ascertain the mechanical resistance offered by small circular aluminum vanes of different diameters, fastened by a small metallic tie-rod to the center of the diaphragm. The small vane was in each case held at its center, coaxial with the diaphragm, and with its plane parallel to that of the diaphragm (see Fig. 23). In changing from one size of vane to another, the mass of the tie-rod was so altered as to maintain the resonant frequency of the loaded diaphragm substantially unchanged, and therefore, likewise, the total equivalent mass. The results are given in the following Table.

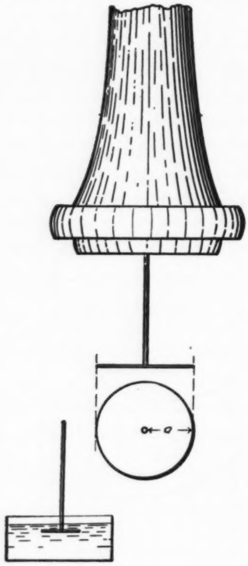


FIG. 23. — AIR RESISTANCE MEASUREMENTS.

TABLE IX.

TABLE OF VANE MECHANICAL RESISTANCE TO VIBRATION.

Test No.	Added Mass of vane & rod gms.	Resonant Frequency cyc/sec. f_0	Vane Area sq. cm. on one surface	Impedance Circle Diam. ohms Z_m	Decrement per sec. Δ	Equivalent Total Mechanical Res. r. dynes/kine	Extra Res.
1	3.00	704	0	26.5	157	1370	—
2	3.11	689	5	21.2	167	1490	120
3	3.18	685	10	17.0	173	1570	200

The equivalent mass of the diaphragm alone was measured independently by the loading method.

It would appear, from the above tests, that the extra mechanical resistance of a circular disk was about 20 dynes/kine for each sq. cm. of disk area,— (πa^2). These results, however, can only be regarded as preliminary.

A few similar measurements, made with circular disk vanes immersed in water instead of in air, gave results of roughly 500 dynes/kine per sq. cm. area of disk, or some 25 times the vibratory resistance of air. In this case it was found that the water not only added mechanical resistance; but also an appreciable extra mass (about 0.5 gm. per sq. cm.) to the vibrating system.

These measurements of extra mechanical resistances are reported, not as definite results, but as indicating the directions in which the motional-impedance circle method may be applied to the analysis of vibrating systems.

In conclusion, the writers desire to express their acknowledgment to the Research Department of the Western Electric Co. for valuable suggestions, help, and special instrument parts.

Summary.

(1) The principal characteristic constants, defining the mechanics of a telephone receiver, are the force-factor A, the equivalent mass m, the equivalent mechanical resistance r and the equivalent elasticity s. They may all be determined from the motional-impedance circle diagram, if one additional independent relation can be secured.

(2) Out of a number of possible additional independent relations, three are considered in detail; namely, the vibrational exploration method for measuring m, the loading of the diaphragm for obtaining m, and the use of an amplitude measurer for determining the maximum cyclic displacement x_m.

(3) While all three of the above-mentioned methods are capable of giving results, the last named is the recommended method. It consists in applying a simple form of amplitude measurer, to the center of the diaphragm, during motional-impedance tests, and observing the amplitude at resonance.

(4) With the amplitude measurer, the characteristic constants are derived for several particular types of telephone receiver tested.

(5) The effects of various influences upon the behavior of a receiver, and on its characteristic constants, are discussed.

Appendix I.

Elementary Theory of Simple Vibration.

We may suppose a material particle of mass m grams at the point P, Figure 24, to rotate in a circular orbit, and in the positive or counter-clockwise direction, about the center O, with uniform angular velocity ω radians per second, as indicated by the arrow. If the radius of the orbit is x cms., we may assume that there is a constant force sx dynes in the direction PO, along the radius vector, so that this centripetal force is proportional to the displacement x from the center O. The vector displacement, measured positively outwards from O at time t seconds, is

$$x = x_0 \, \epsilon^{j\omega t} \qquad \text{cm} \angle \qquad (12)$$

the epoch being selected such that x_0 = OP, when $t = 0$.

The instantaneous velocity of the particle will be directed along the tangent PP^1 at P, and its vector value will be:

$$\dot{x} = j\omega x_0 \, \epsilon^{j\omega t} = j\omega x \qquad \text{cm/sec} \angle \qquad (13)$$

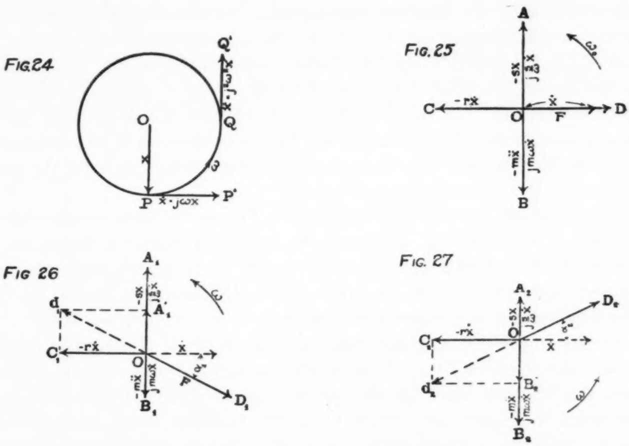
The instantaneous acceleration of the particle will also be directed along the tangent QQ^1 at Q, a point 90° advanced in phase beyond P, and

$$\ddot{x} = (j\omega)^2 \, x_0 \, \epsilon^{j\omega t} = -\,\omega^2 x \qquad \text{cm/sec}^2 \angle \qquad (14)$$

The forces acting on the particle at any instant, such as that indicated in Figure 25, will then be (1) the elastically restoring force $-sx = j\frac{s}{\omega}\dot{x}$ dynes in the direction PO, or opposite to the direction of x. (2) the force opposing the velocity, or $-r\dot{x}$ dynes acting in the direction P^1P or OC, assuming that this force acts in simple proportion to the instantaneous velocity, and (3) the force opposing acceleration, or inertia force, acting in the direction Q^1Q, or OB $= -\,m\ddot{x} = -jm\omega\dot{x}$ dynes. The vector sum of these forces, in conjunction with a rotatory impressed force F dynes along OD, which sustains the motion, must be zero.

The reactive forces (1) and (3) must be in mutual opposition at any instant, because (1) is $j\frac{s}{\omega}\dot{x}$, and the other is $-jm\omega\dot{x}$. Their relative magnitudes, however, depend upon the value of the angular velocity

ω of the particle in the orbit. It is evident that (1) diminishes with increase of ω, while (3) augments. The vector diagram of Figure 25 represents the system of forces acting on the particle for the particular value ω_0, at which the two reactive forces (1) and (3), of resilience and inertia respectively, equate and cancel. The direction of reference OD, or standard phase, is the vector of instantaneous velocity $\dot{x}$, at the moment selected. Then OA represents force (1) to magnitude and phase, or the vector reactive force of resilience tending to bring the particle to the center O, Figure 24. The equal and opposite reactive force OB, of inertia, tends to move it centrifugally away from O,



FIGS. 24, 25, 26, 27. — DIAGRAMS OF MOTIONAL EQUILIBRIUM.

in the direction of instantaneous displacement x, Figure 24. The force OC is directed opposite to the instantaneous velocity. The impressed vector rotating force F = OD, is directed in phase with this velocity. It balances the retarding force OC. The active forces OD and OC therefore cancel, while we have seen that the reactive forces OA and OB also cancel; so that the system will retain a steady state of motion, with the angular velocity ω_0. The power put into the system by the impressed force is $F\dot{x}$ ergs per second, and this is equal to the dissipatory output of the system $r\dot{x}^2$, through the action of the frictionally retarding force.

It is well known that the forces in the simple rectilinear vibratory motion of a particle about a position of rest may be regarded as the projection, upon a line through this position, of the corresponding rotating system in a plane. The vibratory motions and forces of a particle subjected to (1) a simple vibratory resilient force $-sx$ dynes, (2) a motional retarding force $-r\dot{x}$ dynes, and (3) an inertia force $-m\ddot{x}$ dynes, together with (4) a simple harmonic impressed force maintaining the motion, may therefore be considered as the projection of a vector system like that in Figure 25, on the reference line COD, when that system rotates counter-clockwise about the center O, with angular velocity ω radians per second. The instantaneous projection of OB will then be the inertia force opposing acceleration of the particle, for the instant considered, that of OA the instantaneous resilient force, that of OC the instantaneous frictional retarding force, and that of OD the impressed vibratory force, or *vibromotive force* (vmf). The instantaneous velocity will be the projection of OD, when divided by r. The instantaneous displacement will be the projection of OA reversed, when divided by s. The instantaneous acceleration will be the projection of OB reversed, divided by m.

Consequently, for the case indicated in Figure 25, with reactive equilibrium, i. e., equality between the opposing reactive forces $s\dot{x}/\omega$ and $m\omega\dot{x}$, the vibrational velocity $\dot{x}$, will be in phase with the impressed vmf. OD; while the vibrational displacement x will be 90° retarded behind the impressed vmf.

At the impressed angular velocity less than ω_0, of reactive equilibrium, the reactive force (1) of resilience $js\dot{x}/\omega$ dynes, will be greater than the reactive force (3) of inertia $-jm\omega\dot{x}$ dynes. Such a case is indicated in Figure 26; where OA_1, the resilient force, exceeds OB_1 the inertia force. Their difference is OA_1^1, the resultant reactive force. The impressed force $F = OD_1$ must now equilibrate the resultant of OA_1^1 and the motional retarding or frictional force OC_1. It will then be seen that, in the steady state, the velocity $\dot{x}$ leads the impressed force by an angle α_1. The displacement, in line with OB_1, is now less than 90° behind the impressed force.

At any impressed angular velocity greater than that of reactive equilibrium ω_0, the inertia force will overcome the resilient force. This is the case represented in Figure 27; where the vector inertia force OB_2 exceeds the vector resilient force OA_2. Their difference OB_2^1, is the resultant reactive force, which, combined with the frictional force OC_2, gives the resultant force Od_2 to be overcome, or to be equilibrated by the impressed vector force $F = OD_2$, which now leads

the velocity $\dot{x}$, at standard phase, by the angle a_2. The displacement along OB_2 is now more than 90° behind the impressed force.

Consequently, as the angular velocity of constant impressed vmf. increases from zero to infinity, the angular velocity ω commences at 90° lead with respect thereto, and indefinitely small magnitude, later comes in phase, with maximum value, at the angular velocity of resonance, or reactive equilibrium, and ends at 90° lag, again with indefinitely small magnitude.

In Figure 28, the line OX represents the value of r, the resistance to motion, taken along the axis of reals. Xp is taken equal to the reactive difference $j\left(m\omega - \frac{s}{\omega}\right)$, drawn parallel to OY, the axis of imaginaries. The vector Op, which may be called the mechanical impedance, will make an angle a with OX, equal to the phase displacement between the velocity and the impressed vmf.

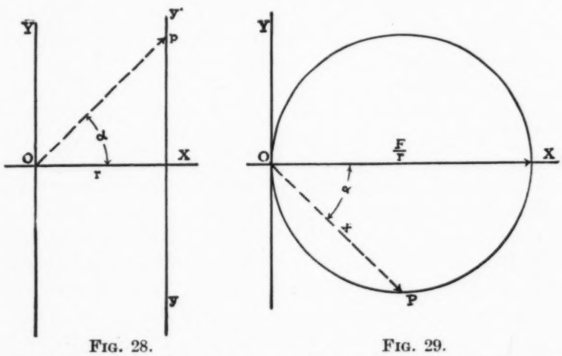


Fig. 28. Fig. 29.

Locus of Mechanical Impedance and Locus of Velocity under Varying Impressed Frequency.

The equilibrium of vector forces represented in Figures 25, 26, and 27, for any steady state of angular velocity, may be expressed algebraically by the formula

$$- sx - r\dot{x} - m\ddot{x} + f = 0 \qquad \text{dynes } \angle \quad (15)$$

or

$$sx + r\dot{x} + m\ddot{x} = f = F\epsilon^{j\omega t} \qquad \text{dynes } \angle \quad (16)$$

Solution in Terms of Velocity.

The solution of the above differential equation, with reference to $\dot{x}$, is

$$\dot{x} = \frac{f}{r + j\left(m\omega - \frac{s}{\omega}\right)} + \dot{x}_1 \epsilon^{-\Delta t + jt\sqrt{\omega_0^2 - \Delta^2}} \qquad \frac{\text{cm}}{\text{sec}} \angle \quad (17)$$

where $\dot{x}_1$ is an initial vector velocity, and $\Delta = r/(2m)$ a numeric, the damping constant per second.

The first term on the right hand of this equation indicates the conditions for the steady state of motion, while the second term is the transient term; i. e., comes into action only during changes of motion in the system, and when f is either varied or withdrawn, the coefficients m, r, and s, remaining constant. We may, therefore, at present consider only constant impressed vmf. and ignore the second or transient term. The formula (17) then reduces to

$$\dot{x} = \frac{F\epsilon^{j\omega t}}{z} = \frac{f}{z} \qquad \frac{\text{cm}}{\text{sec}} \angle \quad (18)$$

where z is the mechanical impedance at the impressed and sustained angular velocity ω. It is evident from Figure 28, that as ω varies, the vector impedance OZ travels over the straight line yXy^1. It is well known that the reciprocal of a variable having a straight-line locus is a variable with a circular locus, so that as ω varies from zero to infinity, the vector locus of $\dot{x}$ will be a circle OXP, Figure 29, with its diameter OX on the axis of reals, and equal in magnitude to F/r cms. per second. That is, the linear velocity of a simple vibratory system, having a retarding force directly proportional to $\dot{x}$, follows a circular locus in regard to magnitude and phase, coming into phase with the impressed force at the resonant angular velocity ω_0 of reactive equilibrium.

Certain relations between the fundamental constants m, r, and s of the vibrator may be determined from the observed distribution of angular velocities around the velocity circle. Thus, the angular velocity of resonance ω_0 is found at the point where the diameter intersects the circle. Since, at resonance, the two reactive forces $\frac{s}{\omega_0} = m\omega_0$, we have

$$\omega_0 = \sqrt{s/m} \qquad \frac{\text{radians}}{\text{sec.}} \quad (19)$$

This gives one relation between s and m, in terms of the observed angular velocity of resonance. Moreover, it can easily be shown that the damping factor

$$\Delta = \frac{r}{2m} = \frac{\omega^2 - \omega_0^2}{2\omega \tan \alpha} \qquad \text{per second} \qquad (20)$$

where ω is an angular velocity on the velocity circle, at a point making an angle α with the diameter. If we select the angular velocity ω_1 at the upper quadrantal point on the circle, for which the impedance angle $\alpha = -45°$ in Figure 28, the above formula becomes

$$\Delta = \frac{\omega_0^2 - \omega_1^2}{2\omega_1} \qquad \text{per second} \qquad (21)$$

Similarly, the angular velocity ω_2 at the lower quadrantal point for which $\alpha = +45°$ gives

$$\Delta = \frac{\omega_2^2 - \omega_0^2}{2\omega_2} \qquad \text{per second} \qquad (22)$$

Summing the last two equations, we obtain

$$\Delta = \frac{\omega_2 - \omega_1}{2} \qquad \text{per second} \qquad (23)$$

so that the damping factor is half the difference between the angular velocities at the quadrantal points. Formulas (20) to (23), or any of them, furnish one relation between r and m, in terms of angular velocities at observed points on the velocity circle. Some third relation is, however, required in order to evaluate m, r, and s.

Transient Motion.

We have seen that the second term of (17) becomes involved at any sudden change or discontinuity in the impressed force f, except in the case when the impressed force f happens to have the resonant angular velocity ω_0. In such a case there is no disturbance in phase relations; but there will be an instantaneous change in the lengths of the vectors OC and OD, Fig. 25, the vectors OA and OB remaining in equilibrium. It seems that in all other cases, a change in the impressed force f must be accompanied by a transient change in the motion of the system, due to the introduction of the second term in

(17). This term is a damped, or logarithmically decaying, periodic velocity, effected not at the angular velocity ω of impressed force, but at the free angular velocity of the system $\omega^1 = \sqrt{\omega_0^2 - \Delta^2} = \omega_0 \sin\gamma$ radians per second, unless Δ is larger than ω_0, in which case the second term of (17) is an ultraperiodic velocity, which may be represented by the projection of a uniformly damped uniform angular velocity in a right hyperbola.[19] For the particular case, $\omega_0 = \Delta$, or the aperiodic case, the motion may be represented by the projection of uniformly damped uniform angular velocity in a parabola. In every case, therefore, the second term of (17) corresponds to the projection of uniformly damped uniform angular velocity in a circle, hyperbola or parabola, i. e., a conic section. In the cases of the telephone-receiver diaphragms examined experimentally, Δ was much less than ω_0; so that only the periodic interpretation of damped uniform angular velocity in a circle needs here to be considered.

Damped Free-Vibration Vector Diagram.

In Figure 30, let a particle at p, of mass m, be attracted centripetally toward the center O, with a force varying directly as the displacement x, and be subjected to a frictional retarding force, oppo-

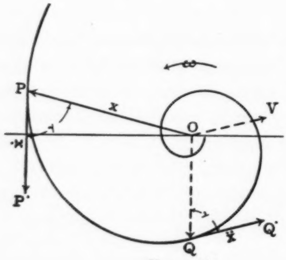


Fig. 30.

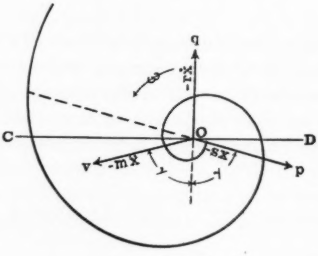


Fig. 31.

Locus of Damped Oscillatory Displacement, Velocity and Acceleration Force.

site to the instantaneous velocity; as well as to an inertia force opposite to the acceleration. Then because there is no impressed force to give energy to the particle, energy will continually be absorbed from it, and

19 Bibliography No. 2 and No. 10.

the orbit of the particle will dwindle, until finally the particle will fall into the center O. This means that the orbit, instead of being a circle, will be an equiangular spiral, in which the tangent PP[1] at any orbital position P, makes an angle of less than 90° with the reversed radius vector PO. The instantaneous acceleration will be directed along the tangent QQ[1] of the spiral at the point Q, (180° − γ°) in advance of P. The centripetal force will be directed along PO, the frictional force in the direction P[1]P, or parallel to QO, and the inertia force in the direction Q[1]Q, or parallel to VO. These conditions are represented in the instantaneous force diagram, Figure 31, where Op is the centripetal force $-sx$, Oq is the frictional force $-r\dot{x}$, and Ov is the inertia force $-m\ddot{x}$, x being the instantaneous displacement:

$$x = x_0\,\epsilon^{(-\Delta + j\omega')t} \qquad \text{cms } \angle \qquad (24)$$

so that

$$r\dot{x} = r\,(-\Delta + j\omega')x_0\,\epsilon^{(-\Delta+j\omega')t} = r\,(-\Delta + j\omega')x \qquad \text{dynes } \angle \qquad (25)$$

and

$$m\ddot{x} = m\,(-\Delta + j\omega')^2\,x_0\,\epsilon^{(-\Delta+j\omega')t} = m\,(-\Delta + j\omega')^2\,x \quad \text{dynes } \angle \qquad (26)$$

For equilibrium we require that [20]

$$-\,sx - r\dot{x} - m\ddot{x} = 0 \qquad \text{dynes } \angle \qquad (27)$$

or

$$-s - r(-\Delta + j\omega') - m\,(-\Delta + j\omega')^2 = 0 \qquad \frac{\text{dynes}}{\text{cm}}\ \angle \qquad (28)$$

whence

$$\omega' = \sqrt{\omega_0^2 - \Delta^2} = \omega_0 \sin\gamma \qquad \frac{\text{radians}}{\text{sec.}} \qquad (29)$$

where

$$\frac{\Delta}{\omega_0} = \cos\gamma \qquad \text{numeric} \qquad (30)$$

Each of the quantities x, $\dot{x}$, $\ddot{x}$, pursues an equiangular spiral around the center 0, or may be considered to pursue a circular path with uniform angular velocity, subject to an independent damping factor $\epsilon^{-\Delta t}$.

In the case of simple rectilinear vibrations, the projections of the spiral motion may be taken on a reference axis COD. The initial value of the velocity $\dot{x}_1$ must be such as meets the physical conditions of the system at the moment of a sudden change in the impressed vibromotive force. After the change, the vibratory motion will be the sum of the two terms in (17) or, the sum of the projections of the

20 Bibliography, No. 8.

respective vectors in Figures 26 and 31, the former being rotated at the impressed angular velocity ω, and the latter at the free angular velocity ω^1. The latter motion, however, speedily expires by damping, leaving the former in the steady state without further interference.

Solution in Terms of Displacement.

We have hitherto considered only the solution of (15) in terms of vibratory velocity $\dot{x}$. We may, however, find the solution in terms of the displacement x, by integrating (17) as follows:

$$x = \left(\frac{1}{j\omega}\right)\frac{F\epsilon^{j\omega t}}{r + j\left(m\omega - \frac{s}{\omega}\right)} + \frac{1}{-\Delta + j\omega'}\dot{x}_1\epsilon^{(-\Delta+j\omega')t} \qquad \text{cm} \angle \quad (31)$$

$$= \left(\frac{1}{j\omega}\right)\frac{f}{z} - \left(\frac{1}{\omega_0}\right)\dot{x}_1\epsilon^{(-\Delta+j\omega')t}\underline{/\gamma} \qquad \text{cm} \angle \quad (32)$$

that is, the vector displacement of steady motion lags 90° in phase behind the vector velocity, and is equal in magnitude to that velocity divided by ω. Also the vector displacement for transient motion lags 180°–γ° behind the vector velocity, and its magnitude is that velocity divided by ω_0.

All of the preceding theory is immediately applicable to the case of a simple alternating-current circuit, containing resistance, inductance, and capacitance in simple series, with an impressed sinusoidal emf. when current strength i is substituted for velocity $\dot{x}$, quantity q for displacement x, inductance L for mass m, elastance S for elastic factor s, and electric resistance R for mechanical resistance r.

Applications of Simple Vibrator Theory to Telephone-Receiver Characteristics.

It can be shown [21] that on the simple vibrator theory of telephone diaphragm vibration, the vibromotive force may be expressed as

$$f_i = \mathrm{A}i \qquad \text{instantaneous dynes} \quad (33)$$

where f_i is the alternating electromagnetic pull exerted on the equiv-

21 Bibliography, No. 11.

alent mass of the diaphragm by the pole or poles of the receiver, when sinusoidal alternating current, $i = I_m \epsilon^{j\omega t}$ absamperes, passes through the winding. A is a constant of the receiver, depending upon its structure. It represents the force exerted on the diaphragm per unit current.

It may be defined by the expressions:—

$$A = \frac{N\mathfrak{B}_0}{\mathfrak{R}} = N\mathfrak{B}_0\mathfrak{P} \qquad \text{dynes/absampere} \quad (34)$$

in a monopolar receiver, and

$$A = \frac{2N\mathfrak{B}_0}{\mathfrak{R}} = 2N\mathfrak{B}_0\mathfrak{P} \qquad \text{dynes/absampere} \quad (35)$$

in a bipolar receiver, where N is the number of turns in the winding, including all coils, $\mathfrak{B}_0$ is the mean flux density in the air-gap at the normal position of rest, due to the permanent-magnet system, in the absence of electric current excitation, and $\mathfrak{R}$ is the reluctance of the magnetic circuit to alternating mmfs. $\mathfrak{P} = 1/\mathfrak{R}$, is the permeance of the circuit to such mmfs. Consequently, from a magnetic point of view, the strength of the receiver is A/N dynes per absampere per turn of exciting winding. This is equal to the product of $\mathfrak{B}_0\mathfrak{P}$, the permanent normal flux-density and the permeance of the a.c. mmf. The maximum cyclic pull on the diaphragm will be $A\,I_m$ dynes, and the rms. pull $A\,I_m/\sqrt{2}$ dynes.

Since, however, owing to the effects of hysteresis and eddy-currents, the alternating flux in the air-gap or gaps of the receiver will lag behind the exciting alternating current by some angle $\beta_1°$, the instantaneous pull will not be in phase with the current, taken at standard phase, but will be expressed by

$$f_i = A.i\,\overline{\diagdown\beta_1} \qquad \text{dynes } \angle \quad (36)$$

Consequently, to current as standard phase, the velocity $\dot{x}$ in (6) will be

$$\dot{x} = \frac{Ai}{z}\overline{\diagdown\beta_1} \qquad \frac{\text{cm}}{\text{sec.}} \angle \quad (37)$$

The alternating emf. induced in the winding by the rate of alteration of air-gap, and flux in the permanent magnetic circuit, will be

$$e_x = A.\dot{x}\,\overline{\diagdown\beta_2} \qquad \text{abvolts } \angle \quad (38)$$

where A is the same force-constant as is defined in (34) and (35); or, substituting from (37):

$$e_x = \frac{A^2 i}{z} \diagdown\overline{\beta_1 + \beta_2} \qquad \text{abvolts } \angle \quad (39)$$

From such experimental observations as have yet been made, it appears that $\beta_1 = \beta_2 = \beta$, say; or that the lag of flux in the a.c. magnetic circuit, associated with the alternation of air-gap reluctance, is nearly the same as that associated in the same magnetic circuit with alternation of current. It is hoped to make further investigations on this point. In any case, however, if we consider that $\beta_1 + \beta_2 = 2\beta$, we have for the motional impedance

$$Z = \frac{e_x}{i} = \frac{A}{i} \dot{x} \diagdown\overline{\beta_2} = \frac{A^2 i}{iz} \diagdown\overline{2\beta} = \frac{A^2}{z} \diagdown\overline{2\beta} \qquad \text{absohms } \angle \quad (40)$$

This is the vector change in the impedance of the receiver winding, occurring at any assigned frequency, between the conditions of free vibration and suppressed vibration. Since in (40) the denominator z, the mechanical impedance, follows a straight-line locus (Fig. 5), as the frequency is varied, the system otherwise remaining constant, the motional impedance Z must follow a circular locus, similar to that of Figure 29, except that the diameter of the circle will be depressed below the axis of reals by the angle 2β. From this circle, corrected when necessary for the effect of change in impressed frequency on A, the natural frequency ω_0 and the damping constant Δ can be determined according to formulas (20) to (23).

Solution for Diaphragm Constants A, m, r and s, when x_m is given.

From (36) and (38), taking current phase as standard,

$$A = \frac{f_i \diagup\beta_1}{i} = \frac{f_m}{I_m} = \frac{e_x \diagup\beta_2}{\dot{x}} = \frac{e_{xm}}{\dot{x}_m} \qquad \frac{\text{dynes}}{\text{absampere}} \quad (41)$$

and from (18), at resonance,

$$f_m = r\dot{x}_m \qquad \text{dynes} \quad (42)$$

where f_m is the maximum cyclic value of the impressed force; and $\dot{x}_m$ the maximum cyclic velocity in phase therewith, also from (40)

$$e_{xm} = I_m Z_m \qquad \text{abvolts} \quad (43)$$

where e_{xm} is the maximum cyclic induced voltage, I_m the maximum cyclic current at resonance and Z_m the maximum motional impedance, or the diameter, in ohms, of the motional impedance circle. Substituting in (41) we have

$$\frac{\dot{x}_m r}{I_m} = \frac{I_m Z_m}{\dot{x}_m} \qquad \frac{\text{dynes}}{\text{absampere}} \qquad (44)$$

or

$$r = \frac{I_m^2 Z_m}{(\dot{x}_m)^2} \qquad \frac{\text{dynes}}{\text{kine}} \qquad (45)$$

and

$$I_m^2 Z_m = (\dot{x}_m)^2 r = P_m \qquad \text{abwatts} \qquad (46)$$

where P_m is the maximum cyclic mechanical power at resonance. In these last equations the phase angles disappear and the magnitudes or vector moduli only are presented. The maximum cyclic velocity has a magnitude represented by

$$\dot{x}_m = \omega_0 x_m \qquad \text{kines} \qquad (47)$$

x_m being the maximum cyclic displacement as measured at the resonant frequency ω_0 over an air-gap i. e., near the center of the diaphragm. Consequently,

$$r = \frac{I_m^2 Z_m}{x_m^2 \omega_0^2} \qquad \frac{\text{dynes}}{\text{kine}} \qquad (48)$$

This determines the value of the mechanical resistance r in terms of measured quantities.

From the motional impedance circle, we can obtain the damping constant $\Delta = r/(2m)$, by any of the formulas (20) to (23); so that

$$m = \frac{r}{2\Delta} \qquad \text{gm} \qquad (49)$$

From the observed resonant angular velocity at the extremity of the motional impedance diameter, we have from (19)

$$s = m\omega_0^2 \qquad \text{dynes/cm} \qquad (49a)$$

Finally, from the relation $Z_m = \dfrac{A^2}{r}$ by (40), we have

$$A = \sqrt{Z_m . r} = \frac{I_m Z_m}{x_m \omega_0} \qquad \frac{\text{dynes}}{\text{absampere}} \qquad (50)$$

which completes the series of constants. The working formulas are thus (48) to (50) inclusive.

Appendix II.

Elementary Theory of Equivalent Mass.

Let it be assumed that a flat diaphragm is clamped tightly around a circular edge of radius a cm., without stretch or tension, that it executes very small sinusoidal vibrations, perpendicularly to its plane, in the fundamental mode of motion, i. e., without either nodal circles or nodal diameters, and that these vibrations may be considered as having a certain amplitude which is a function of the radial distance from the center, and which does not vary appreciably in azimuth around the diaphragm.

Then let

x_r =	the vibration amplitude at radius r	cm.
x_0 =	max. " " " center	cm.
$\dot{x}_r$ =	velocity " radius r	cm/sec.
r =	radius of a given point on the diaphragm	cm.
a =	" " the diaphragm clamping circle	cm.
M =	total mass of the diaphragm within the clamping circle	gm.
ρ' =	superficial density of the diaphragm	gm/cm.^{2}
m =	equivalent mass of the diaphragm	gm.
W_r =	kinetic energy of an annulus	ergs
W =	total " " the diaphragm.	ergs

Then in any steady state of vibration, the kinetic energy of motion of any elementary annulus of radius r and width dr in the diaphragm, will be

$$dW_r = \frac{1}{2}\cdot 2\pi r\rho' (\dot{x}_r)^2 \, dr \qquad \text{ergs} \quad (51)$$

and the total kinetic energy will be

$$W = \int_o^a dW_r = \frac{1}{2}\cdot 2\pi\rho' \cdot \int_o^a (\dot{x}_r)^2 \cdot r \cdot dr \qquad \text{ergs} \quad (52)$$

Also if we define the equivalent mass of the diaphragm as such a mass as, moving with the vibrational velocity at the center of the diaphragm, possesses the same kinetic energy as the whole diaphragm in its distributed amplitudes, we have

$$W = \frac{m}{2}(\dot{x}_0)^2 = \frac{1}{2}\cdot 2\pi\rho' \cdot \int_o^a (\dot{x}_r)^2 \cdot r \cdot dr \qquad \text{ergs} \quad (53)$$

whence

$$m = \frac{2\pi\rho'}{(\dot{x}_0)^2}\int_o^a (\dot{x}_r)^2 \cdot r \cdot dr \qquad \text{gm} \quad (54)$$

On the assumption that the vibrations are simply harmonic, or sinusoidal vibrations, we have the well known condition

$$\frac{\dot{x}_0}{\dot{x}_r} = \frac{x_0}{x_r} \qquad \text{numeric} \quad (55)$$

whence

$$m = \frac{2\pi\rho'}{x_0^2}\int_o^a x_r^2 \cdot r \cdot dr \qquad \text{gm} \quad (56)$$

If the integral in the above expression cannot be computed, after the x_r, r curve or amplitude curve has been drawn, we may approximate to the value of the integral by a process of quadrature. Let Figure 32

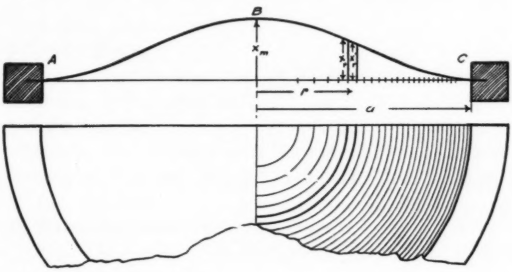


FIG. 32. — ILLUSTRATING METHOD OF DIAPHRAGM EQUIVALENT MASS DETERMINATION FROM EXPLORATION DATA.

represent the amplitude curve, assumed to be known. Let a be divided into a series of n elements, corresponding to successive annuli of equal surface area. Let $x^1{}_r$ be the mean value of the amplitude in any single annulus.

$$\frac{mx_0^2}{2} = \frac{1}{2}\frac{M}{n}\left\{ (x'_1)^2 + (x'_2)^2 + (x'_3)^2 + \ldots\ldots + (x'_n)^2 \right\} \qquad \text{ergs} \quad (57)$$

or

$$m = \frac{M}{n}\frac{\sum (x'_r)^2}{x_0^2} \qquad \text{gm} \quad (58)$$

Consequently, the equivalent mass of such a diaphragm can always be computed from an exploration curve of the amplitude of vibration over its surface.

The ratio $\frac{m}{M}$ may be called the equivalent mass factor.[22]

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List of Symbols Employed.

A	Force factor of an electromagnetic receiver (dynes/absampere).
A_1	" " " a loaded " " " "
a	Radius of a circular disk, or of a diaphragm, within clamping circle (cm).
$\alpha, \alpha_1\ \alpha_2$	Phase angles whose tangents are mechanical reactance factors (radians or degrees).
ab- or abs-	Prefix denoting a C. G. S. magnetic unit.
$\mathfrak{B}_0$	Mean flux density over air-gap or gaps in receivers, due to permanent magnets (gausses).
$\beta = \frac{\beta_1 + \beta_2}{2}$	Mean phase angle of force factor.
$\beta_1\ \beta_2$	Phase lags of force and of induced emf. (radians or degrees).
$\Delta = r/(2m)$	Damping constant of a receiver diaphragm. Numeric per second.
d	Sign of differentiation.
emf.	Contraction for electromotive force.
e_x	Motionally induced emf. in windings, at any instant. (abvolts $\angle$).
$\epsilon = 2.71828$	Napierian base.
e_{xm}	Maximum cyclic value of motional emf. (abvolts).
F F_1 F_2	Maximum cyclic value of a vmf. (dynes).
f	Frequency of alternation. (cycles per second). Also vibromotive force (vmf.) on a diaphragm. (dynes $\angle$).
f_0	Resonant frequency of alternation. (cycles per second).
f_i	Instantaneous value of a vmf. due to an alt. current i absamperes (dynes $\angle$).
f_m	Maximum cyclic value of vmf. (dynes).
γ	Angle of equiangular spiral in free oscillation. (radians or degrees).
I	Root mean square value of sinusoidal alternating current. (absamperes)
I_m	Maximum cyclic value of sinusoidal alt. current at resonance. (absamperes).
i	Instantaneous value of sinusoidal alt. current in winding. (absamperes).
$j = \sqrt{-1}$.	
kine	1 cm. per sec. (unit velocity in C. G. S. system).
L	Inductance of receiver winding damped. (abhenrys).
L^1	" " " " free. (abhenrys).
M	Total active mass of diaphragm within clamping circle. (grams).
m	Equivalent mass of a diaphragm. (grams).
m_1	Mass attached to the center of a diaphragm as load. (grams).
μ	Symbol for 10^{-3} mm., or 1 micron.
N	Number of turns in winding of a receiver, including both poles. (numeric).
n	Number of annuli into which a vibrating diaphragm may be supposed to be divided. (numeric).
P_m	Maximum cyclic mechanical power in diaphragm. (abwatts or ergs/sec).
$\mathfrak{P}$	Permeance of magnetic circuit to alternating mmfs. $(\text{oersted})^{-1}$.
$\pi = 3.14159$	
R	Resistance of receiver winding, diaphragm damped. (absohms or ohms).
R^1	Resistance of receiver winding, diaphragm free (absohms or ohms).

$\Re$	Reluctance of Magnetic Circuit to alternating mmfs. (oersteds).
r	Equivalent motional resistance of a diaphragm. (dynes/kine).
r	In theory of equivalent mass, the radius of a point on diaphragm. (cm).
ρ^1	Superficial density of diaphragm. (gm/sq. cm.)
s	Stiffness constant of a diaphragm. (dynes per cm.)
t	Time elapsed from a certain epoch. (seconds).
vmf.	Contraction for vibromotive force, or alternating mechanical force tending to set up vibration. (dynes $\angle$).
W	Maximum cyclic kinetic energy of diaphragm. (ergs).
W_r	Maximum cyclic kinetic energy of an annulus of width dr, and radius r. (ergs).
X	Reactance of receiver winding, diaphragm damped. (absohms or ohms).
X^1	Reactance of receiver winding, diaphragm free. (absohms or ohms).
XY	Cartesian rectangular coordinates in a plane.
x	Vibratory displacement of a diaphragm, at any time, from its static position of rest. (cm).
x_m	Maximum cyclic vibratory displacement of diaphragm at any frequency. (cm).
x_0	Maximum cyclic vibration amplitude at center of diaphragm. (cm).
$\dot{x}$	Vibratory velocity of a diaphragm at any time. (kines $\angle$).
$\dot{x}_m$	Maximum cyclic vibratory velocity of diaphragm. (kines).
$\ddot{x}$	Vibratory acceleration of a diaphragm at any time. (kines/sec.).
$\dot{x}_1$	Initial velocity of a simple vibrator subject to damping. (kines).
$x^1_1\ x^1_2\ x^1_n$	Maximum cyclic displacements at mid-radii of annular diaphragm segments. (cm).
x_r	Vibrational amplitude of diaphragm at radius r, and any instant (cm).
Z	Motional impedance of a receiver at any frequency. (absohms $\angle$).
$Z_m\ Z_{ms}$	Maximum or diametral amplitude of motional impedance at resonance. (absohms).
$z = r + jz$	Mechanical impedance of diaphragm. (dynes/kine $\angle$).
ω	Angular velocity of vibratory simple-harmonic motion (radians/sec).
ω_0	Angular velocity of vibratory simple-harmonic motion at resonance (radians/sec.).
ω^1	Free angular velocity in presence of damping. (radians/sec.)
ω_{01}	Resonant angular velocity of a loaded diaphragm. (radians/sec.)
$\angle$	Sign of a complex quantity.
$\sim$	Symbol for "cycles-per-second."